Date: 3/18/24

Chp: Chp. 4.3 → Connecting f' & f'' w/ the graph of f

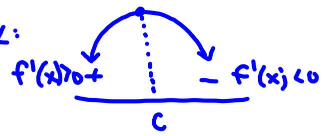
Obj: • Use the First Derivative Test
• Find concavity
• Find pts of Inflection

First Derivative Test

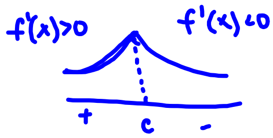
The following test applies to a continuous f(x). At a critical pt, c:

1) If f' changes sign from + to - @c, then the f(x) has a local max @c.

- Ex:



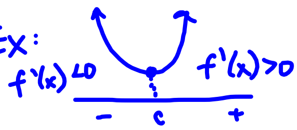
$f'(c)=0$



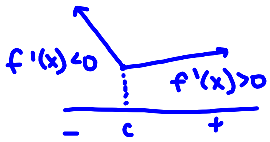
$f'(c)=$ undef.

2) If $f'(x)$ changes sign from - to + @c, then the f(x) has local min @c

- Ex:



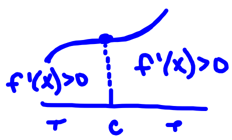
$f'(c)=0$



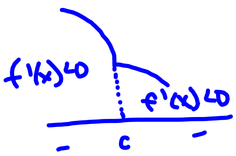
$f'(c)=$ undef.

3) If $f'(x)$ does not have a sign change, then f(x) has no local extrema @c.

- Ex:



$f'(c)=0$



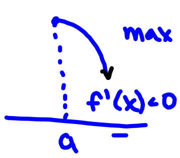
$f'(c)=$ undefined

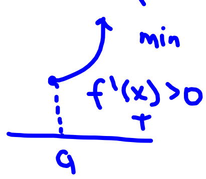
Endpts:

1) If $f'(x)<0$ (or >0) for $x>a$, then the f(x) has a local max (min) @ a.

- Ex:

(left endpt) a

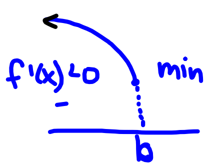


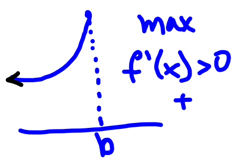


2) right endpt (b)

If $f'(x)<0$ (or >0) for $x<b$, then the function has a local min (max) @ b.

- Ex:





Ex. 1 - Use the FDT to find extrema.

a) $f(x) = x^3 - 12x - 5$

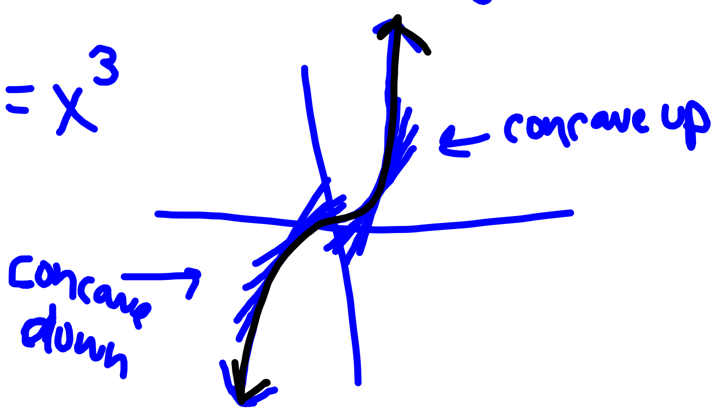
$$f'(x) = 3x^2 - 12$$
$$0 = 3x^2 - 12$$
$$12 = 3x^2$$
$$4 = x^2$$
$$\pm 2 = x$$

b) $g(x) = (x^2 - 3)e^x$

* Concavity

- Concave down = tangents above curve
- Concave up = tangents below curve

- Ex: $y = x^3$



- concave down = if y' is decreasing on interval
- concave up = If y' is increasing on interval.

Concavity Test

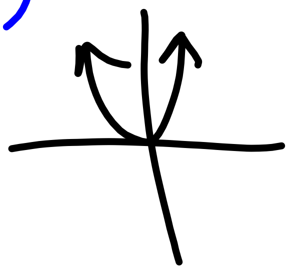
- concave down = if $y'' < 0$
- concave up = if $y'' > 0$

Ex. 2 - Determine concavity.

a) $y = x^2$ on $(3, 10)$

$y' = 2x$

$y'' = 2$ concave up



b) $y = 3 + \sin x$ $(0, 2\pi)$

$y' = \cos x$

$y'' = -\sin x$

$-\sin x > 0$ → $\sin x < 0$

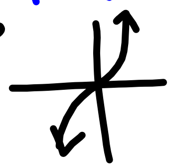
concave up $(\pi, 2\pi)$

$-\sin x < 0$

$\sin x > 0$

$(0, \pi)$ concave down

*Pts of Inflection = pt where the graph of a function has a tangent line & concavity changes. Ex: $y=x^3$



- y'' is + on one side & − on the other
  - when above statement is true then y'' is 0 or undefined
  - $y''=0$ @ a pt of inflection & y' has a local max or min.

To find pts of inflection → Do the 2nd Derivative Test

find the 2nd derivative
Set $=0$ & solve
put on a # line & test w/ 2nd deriv.

Ex. 3 - Find all pts of Inflection

a) $y = e^{-x^2}$

$y' = e^{-x^2} \cdot -2x$

$y' = -2x \cdot e^{-x^2}$

$y'' = -2 \cdot e^{-x^2} + -2xe^{-x^2} \cdot -2x$

$y'' = -2e^{-x^2} + 4x^2 e^{-x^2}$

$0 = -2e^{-x^2} + 4x^2 e^{-x^2}$

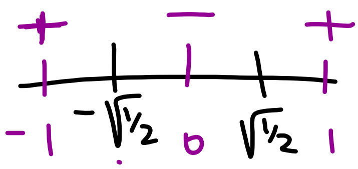
$0 = 2e^{-x^2}(-1 + 2x^2)$

$0 = -1 + 2x^2$

$1 = 2x^2$

$\frac{1}{2} = x^2$

$\pm\sqrt{1/2} = x$



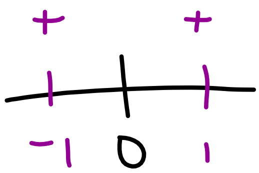
b) $y = x^4$

$y' = 4x^3$

$y'' = 12x^2$

$0 = 12x^2$

$0 = x$



c) $y = \sqrt[3]{x} \to x^{1/3}$

$y' = \frac{1}{3}x^{-2/3}$

$y'' = -\frac{2}{9}x^{-5/3}$

$y'' = \frac{-2}{9(\sqrt[3]{x})^5}$

$0 = \frac{-2}{9(\sqrt[3]{x})^5}$

$0 = -2$ DNE

Ex. 4 - Find the velocity, acceleration, & motion of the function

$$f(x) = 2x^3 - 14x^2 + 22x - 5$$

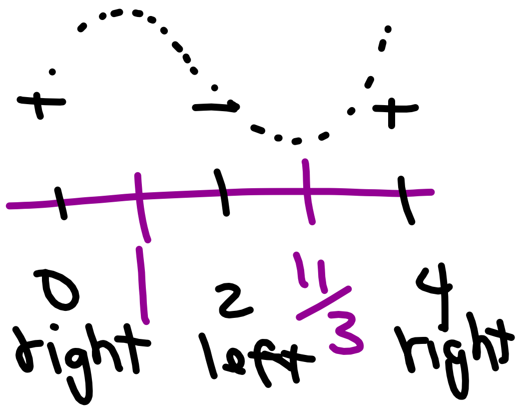
$f'(x) = 6x^2 - 28x + 22 \rightarrow$ velocity

$f''(x) = 12x - 28 \rightarrow$ acceleration

$0 = 6x^2 - 28x + 22$

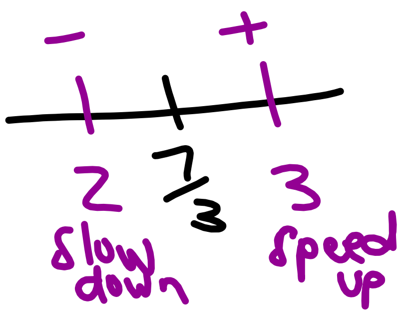
$0 = 2(3x^2 - 14x + 11)$

$x = 1, \frac{11}{3}$



$0 = 12x - 28$

$\frac{7}{3} = x$



<u>Homework:</u>

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