Date: 4/14/25

Chp: Chp 4:6 → Related Rates

Obj: • Solve related rate equations
• Explore solution strategies

* To find a related rate, figure out your formula then find the derivative of the formula (implicit differentiation).

Solution Strategy

1) Understand the problem → identify the rate of change you seek & the rate of change you know

2) Draw a picture → label all important parts & distinguish constants from variables that change over time.

3) Write the equation that relates the variable whose rate of change you seek w/ the variable whose rate of change you know.

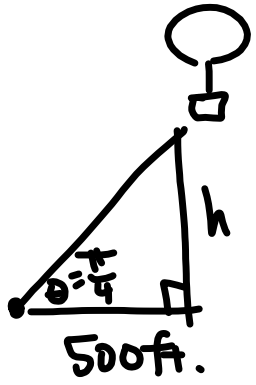
4) Differentiate implicitly (usually in respect to time, t).

5) Substitute in any values that depend on time.

6) Intrepret the solution → label using correct units & ask does it make sense.

Ex. 1 - A hot air balloon rises straight up from a level field & is tracked by a range finder 500 ft. from the lift-off point. At the moment the range finder's ∠ of elevation is $\pi/4$, the angle is increasing at a rate of 0.14 radians per minute. How fast is the balloon rising at that moment?

1) Seek: dh Know: height, $\angle$, $d\theta$ → 500, $\frac{\pi}{4}$, $0.14 \frac{rad}{min}$

2) 

3) $\tan\theta = \frac{h}{500} \rightarrow 500\tan\theta = h$

4) $500\sec^2\theta\, d\theta = dh$

5) $500\sec^2\frac{\pi}{4}(0.14) = dh$

6) 140 ft/min

Ex. 2 – Assume the radius r of a sphere is a differentiable function of t & let V be the volume of the sphere. Find an eq. that relates $\frac{dV}{dt}$ & $\frac{dr}{dt}$.

a) $V = \frac{4}{3}\pi r^3$

$$dV = 4\pi r^2 dr$$

b) Volume of a cone →

$$V = \underbrace{\frac{1}{3}\pi r^2}_{u} \overset{v}{h}$$

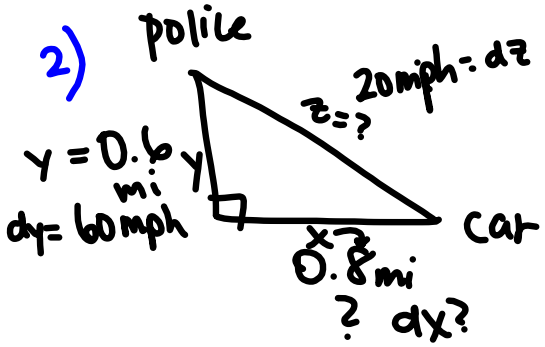
$$dV = \frac{2}{3}\pi r\,dr\,h + 1\,dh \cdot \frac{1}{3}\pi r^2$$

$$dV = \frac{2}{3}\pi r\,dr\,h + \frac{1}{3}\pi r^2\,dh$$

Ex 3 A police car, approaching a right-angled intersection from the north, is chasing another car that has turned the corner due east. When the police car is 0.6 mi north of the intersection & the other car is 0.8 mi east, the police determined the distance between them and the car is increasing @ 20 mph. If the police car is moving @ 60 mph at the instant of measurement, what is the speed of the car?

1) Seek: dx know: y, dy, dz, x

2)



$0.6^2+0.8^2=z^2$

$1=z$

3) $x^2+y^2=z^2$

$2x\,dx+2y\,dy=2z\,dz$

4) $2(0.8)dx+2(0.6)(60)=2(1)(20)$

$1.6\,dx-72=40$

$dx=70$ mph

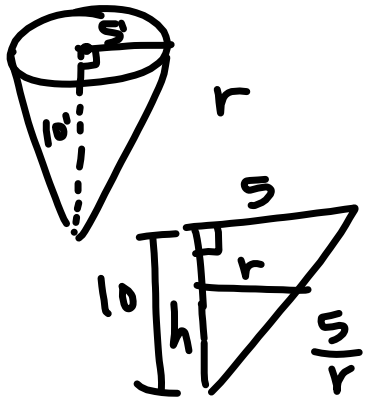
5)

Ex. 4 – Water runs into a conical tank at a rate of 9 ft^3/min. The tank stands point down & has a height of 10 feet with a base radius of 5 ft. How fast is the water level rising when the water is 6 ft deep?

1) Seek: dh

Know: r, h, dV

2)



3) $V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\left(\frac{1}{2}h\right)^2 h$

4) $V = \frac{1}{3}\pi\frac{1}{4}h^2 h$

$V = \frac{1}{12}\pi h^3$

5) $\frac{5}{r} = \frac{10}{h}$

$10r = 5h$

$r = \frac{1}{2}h$

$dV = \frac{1}{4}\pi h^2 dh$

$9 = \frac{1}{4}\pi(6)^2(dh)$

$36 = \pi(36)dh$

$1 = \pi dh$

$\frac{1}{\pi} = dh$

$0.32 = dh$ ft/min

4)

Homework:

p.251 (#1-13 odds)