Date:

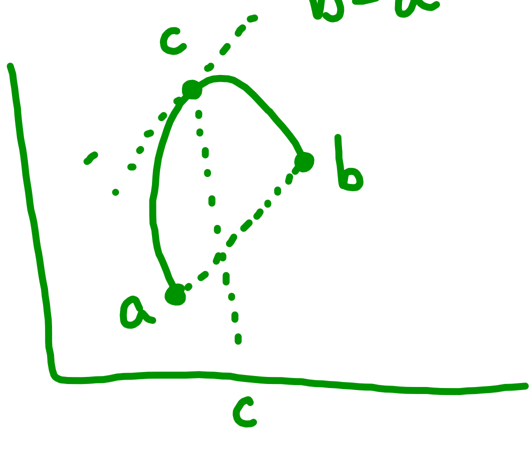
Chp: Chp. 4:2 → Mean Value Thrm

Obj: • Be able to apply the Mean Value Thrm
- Determine if a function is increasing or decreasing

Mean Value Thrm

If the function is continuous @ every pt on $[a, b]$ & differentiable at every pt in the interior (a, b) then there is at least one pt, c, where:

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$



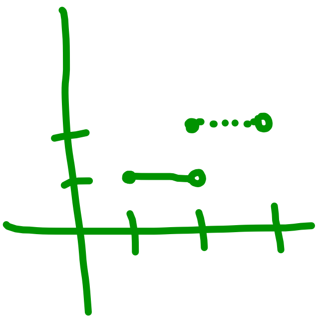
Ex. 1 $y = |x|$ $[-1, 1]$

1) cont? yes

2) diff? No

Ex. 2

$y = [\![x]\!]$ $[1, 2]$



1) Cont? No

Ex.3 – State whether the function satifies the MVT & if so, find c.

a) $f(x)=x^2$ $[0,2]$

- cont? yes
- diff? yes

$f'(x)=2x$

$2x=\frac{0-4}{0-2}$

$2x=2$

$x=1$

b) $f(x)=\sqrt{x^2}+1$ $[-1,1]$

a) cont? yes

b) diff? No

c) $f(x)=\begin{cases} x^3+3 & x<1 \\ x^2+1 & x\geq 1 \end{cases}$ $[-1,1]$

a) cont? No

Ex. 4 - Let $f(x) = \sqrt{1-x^2}$; $A(-1, f(-1))$, $B(1, f(1))$. Find a tangent to $f(x)$ in $(-1, 1)$ that is parallel to secant AB.

a) cont? yes

b) diff? yes

1

$$f(x) = \sqrt{1-x^2} = (1-x^2)^{1/2}$$

$$f'(x) = \frac{1}{2}(1-x^2)^{-1/2}(-2x)$$

$$f'(x) = \frac{-x}{\sqrt{1-x^2}}$$

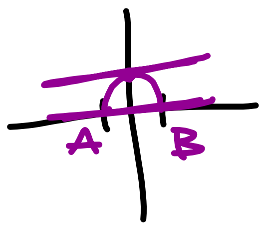
$$\frac{-x}{\sqrt{1-x^2}} = \frac{0-0}{-1-1}$$

$$\frac{-x}{\sqrt{1-x^2}} = 0$$

$$-x = 0$$

$$x = 0$$

$$f'(x) = \frac{-x}{\sqrt{1-x^2}}$$



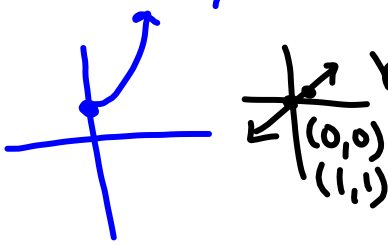
$$f'(0) = \frac{-0}{\sqrt{1-0^2}} = 0$$

$$m = 0$$

$$\cancel{\overset{(1)}{y} = 0\overset{(1)}{x} + b}$$

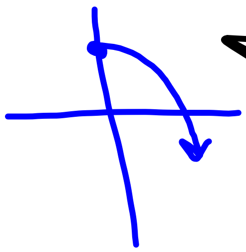
$$y = 1$$

* **Increasing** = $f(x)$ increases on an interval if $x_1 < x_2$ & $f(x_1) < f(x_2)$

Ex: 

risers up to The right

* **Decreasing** = $f(x)$ decreases on an interval if $x_1 < x_2$ & $f(x_1) > f(x_2)$

Ex: 

falls down to the right

Corollary 1

Let $f(x)$ be continuous on $[a,b]$ & differentiable on (a,b)...

1) If $f'(x) > 0$, then the $f(x)$ increases on $[a,b]$

2) If $f'(x) < 0$, then the $f(x)$ decreases on $[a,b]$

3) If $f'(x) = 0$, then the $f(x)$ is constant on $[a,b]$

Ex.5 - Where is $f(x)=x^3-4x$ increasing? decreasing? Extrema?

$f'(x)=3x^2-4$

Inc

$f'(x)>0$

$3x^2-4>0$

$3x^2>4$

$x^2>\frac{4}{3}$

$x>\pm\sqrt{4/3}$

$x>\sqrt{4/3}$ or $x<-\sqrt{4/3}$

Dec

$3x^2-4<0$

$3x^2<4$

$x^2<4/3$

$x<\pm\sqrt{4/3}$

$-\sqrt{4/3}<x<\sqrt{4/3}$

$3x^2-4=0$

$3x^2=4$

$x^2=4/3$

$x=\pm\sqrt{4/3}$

	1.15	-1.15
x	$\sqrt{4/3}$	$-\sqrt{4/3}$
y	-3.08	3.08
	local min	local max

Ex. 6 - Apply the MVT on the interval then find all values c, if possible.

a) $f(x) = x(x^2 - x - 2)$ $\quad x^3 - x^2 - 2x$ $\quad [-1, 1]$

cont? yes
diff? yes

$f'(x) = 3x^2 - 2x - 2$

$$3x^2 - 2x - 2 = \frac{0+2}{-1-1}$$

$$3x^2 - 2x - 2 = -1$$

$$3x^2 - 2x - 1 = 0$$

$$x = \frac{+2 \pm \sqrt{2^2 - 4(3)(-1)}}{2(3)}$$

$$x = \frac{2 \pm 4}{6} = 1, -\frac{1}{3}$$

Ex. 7 - Find all the intervals on which the function is increasing or decreasing

a) $f(x) = 4x^3 - 15x^2 - 18x + 7$

Homework:

p.202 (#1-9 odds, 15-25 odds)