Date: 2/26/24

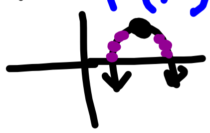
Cnp: Chp 4.1 → Extreme Values of Functions

Obj: • Determine relative & absolute max & min values of a function

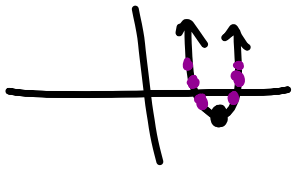
## Absolute Extreme Values

Let $f$ be a function w/ Domain $D$. The $f(c)$ is the....

a) absolute max on $D$ iff $f(x) \le f(c)$ forall $x$ in $D$.

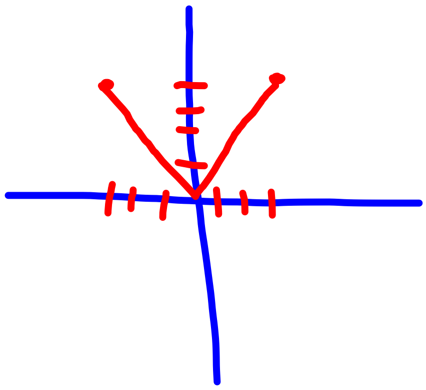


b) absolute min on $D$ iff $f(x) \ge f(c)$ for all $x$ in $D$.



Ex. 1 - Find the extreme values & where they occur.

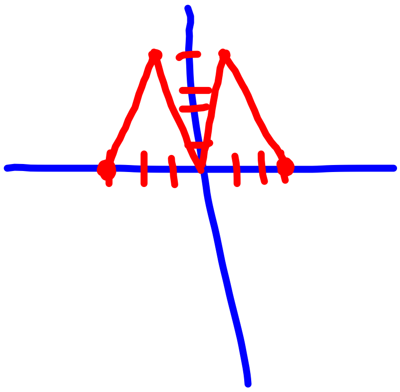
a)



Abs min @ $x=0$

Abs max @ $x=3, -3$

b)



## Local/Relative Extreme Values

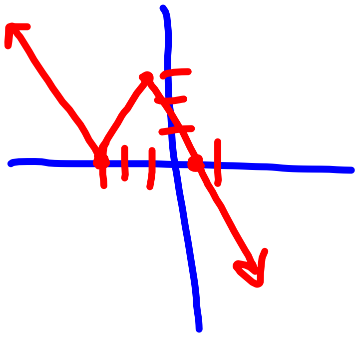
Let $c$ be an interior point of the domain of function $f$. Then $f(c)$ is a ....

a) Local max at $c$ iff $f(x) \leq f(c)$ for all $x$ in some open interval containing $c$.

b) Local min at $c$ iff $f(x) \geq f(c)$ for all $x$ in some open interval containing $c$.

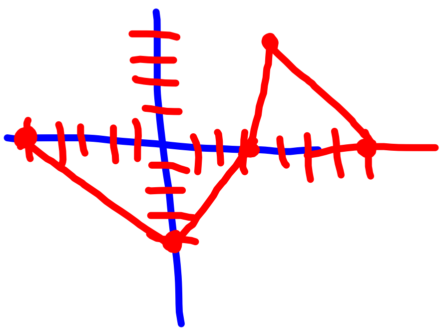
Ex. 2 – Find the extreme values & where they occur.

a)



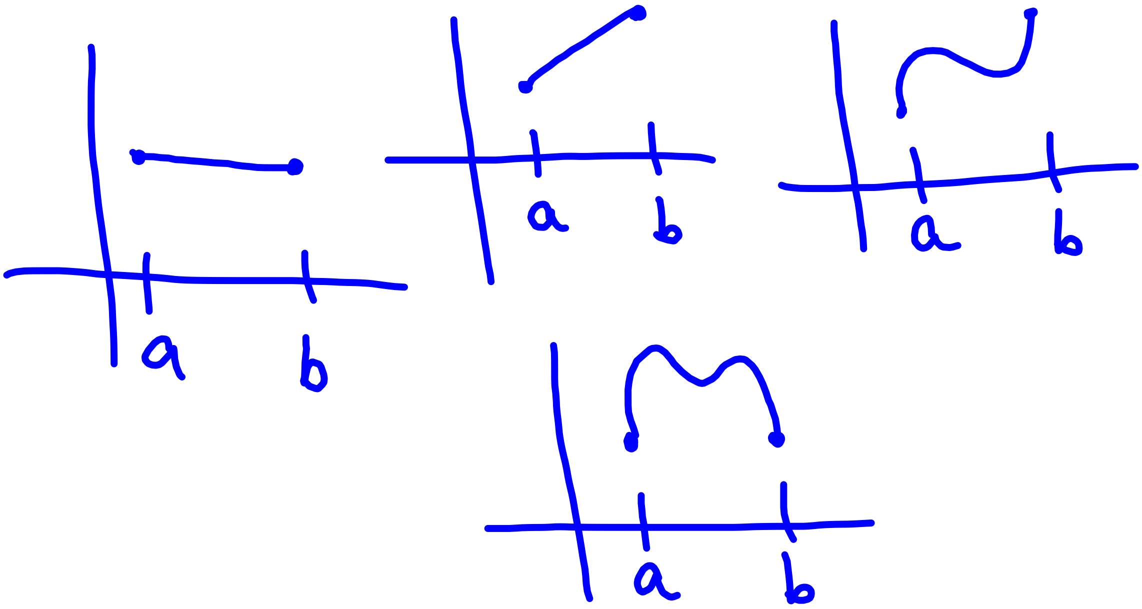
local min @ x = -3
local max @ x = -1

b)



## Extreme Value Thrm

If $f$ is continuous on a closed interval $[a, b]$, then $f$ has both a max & a min value on the interval.




*Extreme values will occur at the endpts or critical points.

* critical pts = x-values when $f'(x)=0$ or DNE!

* relative extrema only occurs @ critical pts. Abs | Global extrema will occur at critical pts. and/or endpts.

A pt in the interior of the domain of a function at which $f'=0$ or $f'$ DNE.

Find the critical point.

Ex: $f(x) = \sqrt{x^2 - 1}$

$f(x) = (x^2 - 1)^{1/2}$

$f'(x) = \frac{1}{2}(x^2 - 1)^{-1/2}(2x)$

$f'(x) = \frac{x}{\sqrt{x^2 - 1}}$

$0 = \frac{x}{\sqrt{x^2 - 1}}$

$0 = x$

Ex. 3c

$$f(x) = \frac{1}{x^2 - 1} \quad (-\infty, -1) \cup (-1, 1) \cup (1, \infty)$$

$$f'(x) = \frac{\cancel{0(x^2-1)} - 2x(1)}{(x^2-1)^2}$$

$$f'(x) = \frac{-2x}{(x^2-1)^2}$$

$$0 = \frac{-2x}{(x^2-1)^2}$$

$$0 = -2x$$

$$0 = x$$

| x | 0 | 0.5 | -0.5 |
|---|---|---|---|
| y | -1 | -1.33 | -1.33 |

local Max

Ex.3d $f(x)=8x^3-3x^2-9x+2 \quad [-1,2]$

$$f'(x)=24x^2-6x-9$$
$$0=24x^2-6x-9$$
$$0=3(8x^2-2x-3)$$
$$0=8x^2-2x-3$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-2)\pm\sqrt{\overset{4+96}{2^2-4(8)(-3)}}}{2(8)}$$

$$\frac{2\pm10}{16} \quad \frac{2+10}{16} \quad \frac{2-10}{16}$$

$$x=\frac{3}{4},\ -\frac{1}{2}$$

| x | $\frac{3}{4}$ | $-\frac{1}{2}$ | -1 | 2 |
|---|---|---|---|---|
| y | -3.96 | 4.75 | 0 | 36 |
| | abs min | local max | local min | abs max |

e) $f(x) = 2\sin x - \cos(2x)$ $[0, 2\pi]$

$f'(x) = 2\cos x - (\cdot\sin(2x))\cdot 2$

$f'(x) = 2\cos x + 2\sin(2x)$

$f'(x) = 2\cos x + 4\sin x\cos x$

$0 = 2\cos x + 4\sin x\cos x$

$0 = 2\cos x(1 + 2\sin x) = 0$ $[0, 2\pi]$

$0 = \cos x$

$\frac{\pi}{2}, \frac{3\pi}{2} = x$

$2\sin x = -1$

$\sin x = \frac{-1}{2}$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$

| x | 0 | $\frac{\pi}{2}$ | $\frac{7\pi}{6}$ | $\frac{3\pi}{2}$ | $\frac{11\pi}{6}$ | $2\pi$ |
|---|---|---|---|---|---|---|
| y | -1 | 3 | -1.5 | -1 | -1.5 | -1 |

Abs Max

Abs. Min

local max

$2\sin x - \cos 2x$

$2\sin 0 - \cos(2 0)$

$2(0) - 1$

$0 - 1$

## Double-Angle Formulas

$$\sin(2u) = 2\sin(u)\cos(u)$$

$$\cos(2u) = 1 - 2\sin^2(u)$$

$$\tan(2u) = \frac{2\tan(u)}{1-\tan^2(u)}$$