9.3 Solving Rational Equations

Essential Question: What methods are there for solving rational equations?

Resource Locker

Explore Solving Rational Equations Graphically

A rational equation is an equation that contains one or more rational expressions. The time t in hours it takes to travel d miles can be determined by using the equation $t = \frac{d}{r}$, where r is the average rate of speed. This equation is an example of a rational equation. One method to solving rational equations is by graphing.

Solve the rational equation $\frac{x}{x-3} = 2$ by graphing.

(A) First, identify any excluded values. A number is an excluded value of a rational expression if substituting the number into the expression results in a division by 0, which is undefined. Solve $x - 3 = 0$ for x.

$x - 3 = 0$

$x =$? **3**

(B) So, 3 is an excluded value of the rational equation. Rewrite the equation with 0 on one side.

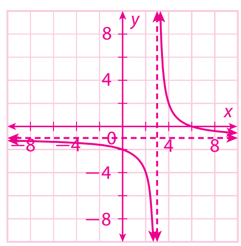
$\frac{x}{x-3} = 2$

$\frac{x}{x-3} - 2$? $= 0$

(C) Graph the left side of the equation as a function. Substitute y for 0 and complete the table below.

	x	y	(x, y)	
−2	0	?	?	**(0, −2)**
−2.5	1	?	?	**(1, −2.5)**
−4	2	?	?	**(2, −4)**
2	4	?	?	**(4, 2)**
0.5	5	?	?	**(5, 0.5)**
−0.5	9	?	?	**(9, −0.5)**

(D) Use the table to graph the function.



(E) Identify any x-intercepts of the graph.

There is an x-intercept at ? . **(6, 0)**

(F) Is the value of x an excluded value? What is the solution of $\frac{x}{x-3} = 2$? ?

No, $x = 6$ is not an excluded value. The solution of $\frac{x}{x-3} = 2$ is $x = 6$.

Professional Development

Integrate Math Processes and Practices

This lesson provides an opportunity to address Math Process and Practice **MPP3**, which calls for students to use and evaluate logical reasoning. Students learn that they can solve rational equations graphically, by writing a related function and finding the zeros of the function. They also learn to solve rational equations algebraically, multiplying the equation by the LCD and converting it into an equivalent polynomial equation. Students draw connections between the excluded values of the expressions and the extraneous solutions of the equation.

Solving Rational Equations

Learning Objective

Students will solve rational equations in one variable, identify extraneous solutions, and solve real-world problems using rational equations.

Math Processes and Practices

MPP3 Using and Evaluating Logical Reasoning

Language Objective

Describe to a partner how to solve rational equations.

Online Resources

An extra example for each Explain section is available online.

Engage

Essential Question: What methods are there for solving rational equations?

Rational equations can be solved algebraically by finding the LCM of the denominators of the rational expressions and multiplying each side of the equation by that LCM. When the equation is simplified, the result is a polynomial equation that can be solved by factoring, graphing, and other methods. Rational equations also can be solved by graphing. For example, each side of a rational equation may be entered as a function. The solution is the x-coordinate(s) of the point(s) at which the graphs intersect.

Preview: Lesson Performance Task

View the Engage section online. Discuss the photo and ask students to list the information they would need to calculate the profit per item. Then preview the Lesson Performance Task.

Explore

Solving Rational Equations Graphically

Integrate Technology

Students have the option of completing the Explore activity either in the book or online.

Integrate Math Processes and Practices

Focus on Problem Solving

MPP1 Students should recognize that the x-intercepts represent the solutions of the equation because they are the zeros of the related function. Help them to make this connection, emphasizing that a point with coordinates $(x, 0)$ represents a value of x for which the function is equal to 0.

Questioning Strategies

If the two sides of the equation $\frac{x}{x-3} = 2$ are graphed as separate functions, will the graphs intersect, and if so, where? Yes; the graphs will intersect at the point (6, 2) whose x-value is the solution of the equation.

Explain 1

Solving Rational Equations Algebraically

Questioning Strategies

How does finding the LCD of the rational expressions and multiplying each side of the equation by the LCD turn the rational equation into a polynomial equation? After the multiplication of both sides by the LCD has been carried out, common factors in the numerators and denominators can be divided out. The resulting denominators on each side are 1, and since the numerators are polynomials, the equation becomes a polynomial equation.

Reflect

1. **Rewriting the equation with 0 on one side helps because the expression on the other side can be graphed and the solution is the x-intercept.**

1. **Discussion** Why does rewriting a rational equation with 0 on one side help with solving the equation?

Explain 1 Solving Rational Equations Algebraically

Rational equations can be solved algebraically by multiplying through by the LCD and solving the resulting polynomial equation. However, this eliminates the information about the excluded values of the original equation. Sometimes an excluded value of the original equation is a solution of the polynomial equation, and in this case the excluded value will be an **extraneous solution** of the polynomial equation. Extraneous solutions are not solutions of an equation.

Example 1 Solve each rational equation algebraically.

(A) $\frac{3x+7}{x-5} = \frac{5x+17}{2x-10}$

Identify any excluded values.

$x - 5 = 0 \qquad 2x - 10 = 0$

$x = 5 \qquad x = 5$

The excluded value is 5.

Identify the LCD by finding all factors of the denominators.

$2x - 10 = 2(x-5)$

The different factors are 2 and $x - 5$.

The LCD is $2(x-5)$.

Multiply each term by the LCD. $\frac{3x+7}{x-5} \cdot 2(x-5) = \frac{5x+17}{2(x-5)} \cdot 2(x-5)$

Divide out common factors. $\frac{3x+7}{\cancel{x-5}} \cdot 2\cancel{(x-5)} = \frac{5x+17}{\cancel{2}\cancel{(x-5)}} \cdot \cancel{2}\cancel{(x-5)}$

Simplify. $(3x+7)2 = 5x + 17$

Use the Distributive Property. $6x + 14 = 5x + 17$

Solve for x. $x + 14 = 17$

$x = 3$

The solution $x = 3$ is not an excluded value. So, $x = 3$ is the solution of the equation.

(B) $\frac{2x-9}{x-7} + \frac{x}{2} = \frac{5}{x-7}$

Identify any excluded values.

$x - 7 = 0$

$x = \boxed{7}$

The excluded value is 7.

Identify the LCD.

The different factors are 2 and $x - 7$.

Collaborative Learning

Small Group Activity

Have students work in groups of 3–4 students. Provide each group with a different rational equation to solve. Instruct students to create a poster showing the equation solved using three different methods: graphically using one function, graphically using two functions, and algebraically. Remind students to look for and to indicate extraneous solutions. Have students share their posters with the class.

The LCD is $2(x-7)$.

Multiply each term by the LCD. $\frac{2x-9}{x-7}\cdot\boxed{2(x-7)}+\frac{x}{2}\cdot\boxed{2(x-7)}=\frac{5}{x-7}\cdot\boxed{2(x-7)}$

Divide out common factors. $\frac{2x-9}{\cancel{x-7}}\cdot\boxed{2(\cancel{x-7})}+\frac{x}{\cancel{2}}\cdot\boxed{\cancel{2}(x-7)}=\frac{5}{\cancel{x-7}}\cdot\boxed{2(\cancel{x-7})}$

Simplify. $\boxed{2}(2x-9)+x\left(\boxed{x-7}\right)=5\left(\boxed{2}\right)$

Use the Distributive Property. $\boxed{4x-18}+x^2-7x=\boxed{10}$

Write in standard form. $\boxed{x^2-3x-28}=0$

Factor. $\left(\boxed{x-7}\right)\left(\boxed{x+4}\right)=0$

Use the Zero Product Property. $x-7=0$ or $\boxed{x+4}=0$

Solve for x. $x=7$ or $x=\boxed{-4}$

The solution $x=\boxed{7}$ is extraneous because it is an excluded value. The only solution is $x=\boxed{-4}$.

Your Turn

Solve each rational equation algebraically.

2. $\frac{8}{x+3}=\frac{x+1}{x+6}$ **The solutions are $x=9$ or $x=-5$.**

Explain 2 Solving a Real-world Problem with a Rational Equation

Rational equations are used to model real-world situations. These equations can be solved algebraically.

Example 2 **Use a rational equation to solve the problem.**

(A) Kelsey is kayaking on a river. She travels 5 miles upstream and 5 miles downstream in a total of 6 hours. In still water, Kelsey can travel at an average speed of 3 miles per hour. What is the average speed of the river's current?

Analyze Information

Identify the important information:

- The answer will be the average speed of the current.
- Kelsey spends 6 hours kayaking.
- She travels 5 miles upstream and 5 miles downstream.
- Her average speed in still water is 3 miles per hour.

Differentiate Instruction

Multiple Representations

Point out to students that they can use the table feature of a graphing calculator to check their solutions to rational equations. They can enter the left side of the equation as Y1 and the right side as Y2. Then, by using the table feature, they can verify that Y1 and Y2 have the same values of x that they determined by using algebra.

Connect Vocabulary EL

Have students look up the definition of *extraneous* and relate its use in this context to its use in non-mathematical contexts.

Questioning Strategies

How can you tell whether a solution of a rational equation is extraneous? It is extraneous if it is an excluded value for one of the rational expressions in the equation.

Integrate Technology

Students can use the graphing method presented in the Explore to check their solutions. Have them graph the corresponding function and compare its zeros to the solutions found algebraically.

Explain 2

Solving a Real-World Problem with a Rational Equation

Avoid Common Errors

Students may need to be reminded to check to see whether the solutions of their equations are extraneous solutions. Remind them to also consider restrictions that are imposed on the variable due to the context of the application.

Questioning Strategies

How could you solve the problem graphically? Graph each side of the equation as a function and find the points of intersection of the two graphs. The x-values of the points of intersection will be the solutions of the equation.

Integrate Math Processes and Practices
Focus on Mathematical Modeling

MPP4 Discuss with students how the rational expressions used in the example model the situation. Have students describe what each numerator and denominator represents, and why the equation models the real-world relationship described in the problem.

Elaborate

Integrate Math Processes and Practices
Focus on Using and Evaluating Logical Reasoning

MPP3 Discuss with students how the algebraic process of converting a rational equation into a polynomial equation can introduce extraneous solutions. Lead them to recognize that polynomial expressions themselves do not have excluded values and, therefore, the solutions to a polynomial equation will all satisfy the polynomial equation. However, if the polynomial equation is derived from a rational equation, any excluded values of the rational equation that are solutions of the polynomial equation will be extraneous.

Summarize The Lesson

How can you use the LCD of the rational expressions in an equation to solve the equation? You can multiply each term in the equation by the LCD. This will change the rational equation into a polynomial equation. You can then solve the polynomial equation, being careful to check whether any of the solutions are extraneous.

Formulate a Plan

Let c represent the speed of the current in miles per hour. When Kelsey is going upstream, her speed is equal to her speed in still water minus c. When Kelsey is going downstream, her speed is equal to her speed in still water plus c.
The variable c is restricted to positive real numbers.
Complete the table.

	Distance (mi)	Average speed (mi/h)	Time (h)
Upstream	5	$3 - c$	$\frac{5}{3-c}$
Downstream	5	$3 + c$	$\frac{5}{3+c}$

Use the results from the table to write an equation.
total time = time upstream + time downstream

$$6 = \boxed{\frac{5}{3-c}} + \boxed{\frac{5}{3+c}}$$

Solve

$3 - c = 0 \qquad 3 + c = 0$

$\boxed{3} = c \qquad c = \boxed{-3}$

Excluded values: 3 and -3
LCD: $(3-c)(3+c)$

Multiply by the LCD. $\quad 6 \cdot \boxed{(3-c)(3+c)} = \frac{5}{3-c} \cdot \boxed{(3-c)(3+c)} + \frac{5}{3+c} \cdot \boxed{(3-c)(3+c)}$

Divide out common factors. $\quad 6 \cdot \boxed{(3-c)(3+c)} = \frac{5}{\cancel{3-c}} \cdot \boxed{\cancel{(3-c)}(3+c)} + \frac{5}{\cancel{3+c}} \cdot \boxed{(3-c)\cancel{(3+c)}}$

Simplify. $\quad 6 \cdot \boxed{(3-c)(3+c)} = 5 \cdot \boxed{3+c} + 5 \cdot \boxed{3-c}$

Use the Distributive Property. $\quad \boxed{54 - 6c^2} = 15 + 5c + \boxed{15 - 5c}$

Write in standard form. $\quad 0 = \boxed{6c^2 - 24}$

Factor. $\quad 0 = 6(c+2)\left(\boxed{c-2}\right)$

Use the Zero Product Property. $\quad c + 2 = 0$ or $\boxed{c-2} = 0$

Solve for c. $\quad c = \boxed{-2}$ or $c = \boxed{2}$

There are no extraneous solutions. The solutions are $c = -2$ or $c = 2$.

Justify and Evaluate

The solution $c = \boxed{-2}$ is unreasonable because the speed of the current cannot be negative, but the solution $c = \boxed{2}$ is reasonable because the speed of the

Language Support EL

Communicate Math

Have students work in pairs to complete a chart shown on solving rational equations.

Type of Equation	Type of Solution	Notes Explaining Steps
Rational: Write an equation	Graphical	For example, rewrite with 0 on one side and graph the function on the other side.
Rational: Write an equation	Algebraic	For example, find the LCD by factoring, etc.

current can be positive. If the speed of the current is 2 miles per hour, it would take Kelsey 5 hour(s) to go upstream and 1 hour(s) to go downstream, which is a total of 6 hours.

Reflect

3. Why does the domain of the variable have to be restricted in real-world problems that can be modeled with a rational equation?
The variable must make sense in a real-world context. The speed of the current cannot be negative or 0, so the domain of c had to be restricted to positive real numbers.

Your Turn

4. Kevin can clean a large aquarium tank in about 7 hours. When Kevin and Lara work together, they can clean the tank in 4 hours. Write and solve a rational equation to determine how long, to the nearest tenth of an hour, it would take Lara to clean the tank if she works by herself. Explain whether the answer is reasonable.
It would take Lara about 9 hours and 20 minutes to clean the tank by herself. The answer is reasonable because $9\frac{1}{3}$ is positive and $\frac{1}{7}(4) + \frac{1}{9\frac{1}{3}}(4) = 1$.

Elaborate

5. Why is it important to check solutions to rational equations? **To make sure that there are no extraneous solutions.**

6. Why can extraneous solutions to rational equations exist? **See margin.**

7. **Essential Question Check In** How can you solve a rational equation without graphing? **See margin.**

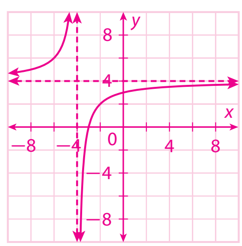
Evaluate: Homework and Practice

Personal Math Trainer
- Online Homework
- Hints and Help
- Extra Practice

Solve each rational equation by graphing using a table of values.

1. $\frac{x}{x+4} = -3$

	x	y	(x, y)	
5	−8	?	?	**(−8, 5)**
6	−6	?	?	**(−6, 6)**
8	−5	?	?	**(−5, 8)**
−4	−3.5	?	?	**(−3.5, −4)**
2	−2	?	?	**(−2, 2)**
3	0	?	?	**(0, 3)**



The x-intercept is at $(-3, 0)$. The solution is $x = -3$.

Evaluate

Personal Math Trainer

Assignment Guide

Level	Concepts and Skills	Practice
Basic	Explore	Exercises 1–2
	Example 1	Exercises 3–6
	Example 2	Exercises 9–12
	H.O.T.	Exercise 28
Average	Explore	Exercise 2
	Example 1	Exercises 4–7
	Example 2	Exercises 11–14
	H.O.T.	Exercises 20–21
Advanced	Explore	Exercise 2
	Example 1	Exercises 5–8
	Example 2	Exercises 13–16
	H.O.T.	Exercises 18–21

Real World Problems

Avoid Common Errors

If students do not obtain an appropriate graph, they may have made errors either in entering the function (or functions) or in choosing an appropriate viewing window. The most common error in entering a rational function is to omit the parentheses around the complete numerator and the complete denominator. Check students' calculators to catch errors and guide them in correcting any errors.

Answers

6. Because the process of multiplying through by the LCD may eliminate the information about excluded values.

7. Rational equations can be solved algebraically by finding the LCM of the denominators of the rational expressions and multiplying each side of the equation by that LCM. When the equation is simplified, the result is a polynomial equation that can be solved.

Exercise	Depth of Knowledge (D.O.K.)	Math Processes and Practices
1–2	**2** Skills/Concepts	**MPP6** Using Precise Mathematical Language
3–8	**1** Recall of Information	**MPP2** Abstract and Quantitative Reasoning
9–16	**2** Skills/Concepts	**MPP4** Mathematical Modeling
17	**1** Recall of Information	**MPP2** Abstract and Quantitative Reasoning
18	**3** Strategic Thinking H.O.T.	**MPP2** Abstract and Quantitative Reasoning
19	**3** Strategic Thinking H.O.T.	**MPP6** Using Precise Mathematical Language

Critical Thinking

Ask students to consider whether it is possible to solve a rational equation by multiplying each of its terms by a common denominator that is not the *least* common denominator of the rational expressions. Have them predict how the result of doing this would compare to using the LCD, and test their predictions using one or more of the exercises.

Integrate Math Processes and Practices
Focus on Abstract and Quantitative Reasoning

MPP2 Students can check their solutions by substituting the values into the original equations and verifying that they make the equation true. Students should observe that any extraneous solutions would make one or more of the rational expressions undefined.

Questioning Strategies

How is the LCD of two rational expressions related to each denominator? It is a multiple of each of the denominators.

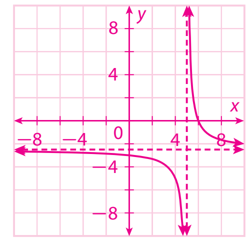
When is the LCD of two rational expressions the product of the denominators? When the denominators have no common factors.

Communicating Math

Call upon students to present their solutions to the class. Ask them to describe how they determined the LCD and how they transformed the rational equation into a polynomial equation. Have them describe the steps of the solution process, and tell how they identified any extraneous solutions.

2. $\dfrac{x}{2x-10}=3$

	x	y	(x, y)	
-3	0	?	?	$(0, -3)$
-3.75	3	?	?	$(3, -3.75)$
-5	4	?	?	$(4, -5)$
2.5	5.5	?	?	$(5.5, 2.5)$
-1.25	7	?	?	$(7, -1.25)$
-2	10	?	?	$(10, -2)$



The x-intercept is at $(6, 0)$. The solution is $x = 6$.

Solve each rational equation algebraically.

3. $\dfrac{9}{4x}-\dfrac{5}{6}=-\dfrac{13}{12x}$ $x = 4$

4. $\dfrac{3}{x+1}+\dfrac{2}{7}=2$ $x=\dfrac{3}{4}$

5. $\dfrac{56}{x^2-2x-15}-\dfrac{6}{x+3}=\dfrac{7}{x-5}$
There is no solution.

6. $\dfrac{x^2-29}{x^2-10x+21}=\dfrac{6}{x-7}+\dfrac{5}{x-3}$
$x = 8$; $x = 3$ is extraneous.

7. $\dfrac{5}{2x+6}-\dfrac{1}{6}=\dfrac{2}{x+4}$ $x = 2$ or $x = -6$

8. $\dfrac{5}{x^2-3x+2}-\dfrac{1}{x-2}=\dfrac{x+6}{3x-3}$
$x = -10$ or $x = 3$

For 9 and 10, write a rational equation for each real-world application. Do not solve.

9. A save percentage in lacrosse is found by dividing the number of saves by the number of shots faced. A lacrosse goalie saved 9 of 12 shots. How many additional consecutive saves s must the goalie make to raise his save percentage to 0.850?

$\dfrac{9+s}{12+s}=0.850$

10. Jake can mulch a garden in 30 minutes. Together, Jake and Ross can mulch the same garden in 16 minutes. How much time t, in minutes, will it take Ross to mulch the garden when working alone?

$\dfrac{1}{30}(16)+\dfrac{1}{t}(16)=1$

11. **Geometry** A new ice skating rink will be approximately rectangular in shape and will have an area of 18,000 square feet. Using an equation for the perimeter P, of the skating rink in terms of its width W, what are the dimensions of the skating rink if the perimeter is 580 feet? The dimensions are 200 feet by 90 feet.

Exercise	Depth of Knowledge (D.O.K.)	Math Processes and Practices
20	**3** Strategic Thinking H.O.T.	**MPP3** Using and Evaluating Logical Reasoning
21	**3** Strategic Thinking H.O.T.	**MPP5** Using Mathematical Tools

12. Water flowing through both a small pipe and a large pipe can fill a water tank in 9 hours. Water flowing through the large pipe alone can fill the tank in 17 hours. Write an equation that can be used to find the amount of time t, in hours, it would take to fill the tank using only the small pipe. $\frac{1}{17}(9) + \frac{1}{t}(9) = 1$

13. A riverboat travels at an average of 14 km per hour in still water. The riverboat travels 110 km east up the Ohio River and 110 km west down the same river in a total of 17.5 hours. To the nearest tenth of a kilometer per hour, what was the speed of the current of the river?

The the speed of the current is about 4.5 km/h.

14. The number of hits must be a whole number. The player would need 12 consecutive hits.

14. A baseball player's batting average is equal to the number of hits divided by the number of at bats. A professional player had 139 hits in 515 at bats in 2012 and 167 hits in 584 at bats in 2013. Write and solve an equation to find how many additional consecutive hits h the batter would have needed to raise his batting average in 2012 to be at least equal to his average in 2013.

15. The time required to deliver and install a computer network at a customer's location is $t = 5 + \frac{2d}{r}$, where t is time in hours, d is the distance (in miles) from the warehouse to the customer's location, and r is the average speed of the delivery truck. If it takes 8.2 hours for an employee to deliver and install a network for a customer located 80 miles from the warehouse, what is the average speed of the delivery truck? **See below.**

16. **Art** A glassblower can produce several sets of simple glasses in about 3 hours. When the glassblower works with an apprentice, the job takes about 2 hours. How long would it take the apprentice to make the same number of sets of glasses when working alone?

It would take the apprentice about 6 hours.

17. Which of the following equations have at least two excluded values? Select all that apply. **B; E**

A. $\frac{3}{x} + \frac{1}{5x} = 1$

B. $\frac{x-4}{x-2} + \frac{3}{x} = \frac{5}{6}$

C. $\frac{x}{x-6} + 1 = \frac{5}{2x-12}$

D. $\frac{2x-3}{x^2-10x+25} + \frac{3}{7} = \frac{1}{x-5}$

E. $\frac{7}{x+2} + \frac{3x-4}{x^2+5x+6} = 9$

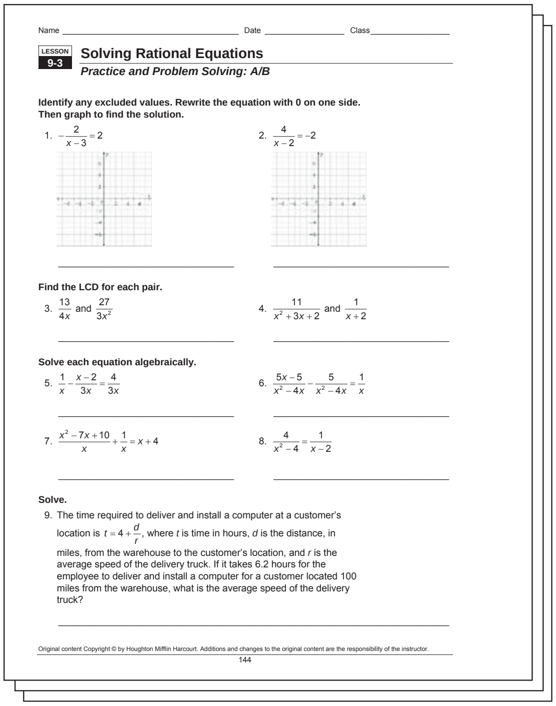
15. The average speed of the truck is 50 mph.

Avoid Common Errors

When solving a rational equation that models a real-world situation, students may erroneously believe that any negative solution must be an extraneous solution. Help them to see that this is not necessarily the case, and remind them to reflect upon the rational expressions contained in the original equation and their excluded values.

Online Resources

- Practice and Problem Solving (three forms)
- Reteach
- Reading Strategies
- Success for English Learners



Name ______ Date ______ Class ______

LESSON 9-3 **Solving Rational Equations**

Practice and Problem Solving: A/B

Identify any excluded values. Rewrite the equation with 0 on one side. Then graph to find the solution.

1. $-\frac{2}{x-3} = 2$

2. $\frac{4}{x-2} = -2$

Find the LCD for each pair.

3. $\frac{13}{4x}$ and $\frac{27}{3x^2}$

4. $\frac{11}{x^2+3x+2}$ and $\frac{1}{x+2}$

Solve each equation algebraically.

5. $\frac{1}{x} - \frac{x-2}{3x} = \frac{4}{3x}$

6. $\frac{5x-5}{x^2-4x} - \frac{5}{x^2-4x} = \frac{1}{x}$

7. $\frac{x^2-7x+10}{x} + \frac{1}{x} = x+4$

8. $\frac{4}{x^2-4} = \frac{1}{x-2}$

Solve.

9. The time required to deliver and install a computer at a customer's location is $t = 4 + \frac{d}{r}$, where t is time in hours, d is the distance, in miles, from the warehouse to the customer's location, and r is the average speed of the delivery truck. If it takes 6.2 hours for the employee to deliver and install a computer for a customer located 100 miles from the warehouse, what is the average speed of the delivery truck?

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Journal

Have students make a list of the steps involved in solving a rational equation algebraically.

Connect Vocabulary EL

Students may not understand the concept of *profit*. Explain that the money you receive when selling an item is called the *sales income,* but this is not the same thing as profit. To calculate the profit, first find how much it *costs* to produce the item. The *profit* is the money left over after subtracting the costs from the income.

Integrate Math Processes and Practices
Focus on Abstract and Quantitative Reasoning

MPP2 Have students graph the profit per item $P_{PI}(d)$ for the number of DVDs sold, d. Have them discuss the maximum profit Kasey can make and what limits the profit. Have students explain how the profit per item changes as the number of items sold increases. The profit will approach \$8.60 per item, but it will never reach it because the cost of materials per DVD must be subtracted.

Answers

18. 0, 1, or 2; the equation becomes $x^2 - bcx + ab = 0$ after multiplying by the LCD and putting the equation in standard form. A quadratic equation can have 0, 1, or 2 solutions. The number of solutions of this equation depends on the values of a, b, and c.

19. Possible answer: The graph of $y = \frac{(x-4)(x-3)}{x-4} = \frac{x^2 - 7x + 12}{x-4}$ has the same graph as $y = x - 3$ but has an open circle at $x = 4$.

Lesson Performance Task Scoring Rubric

Points	Criteria
2	Student correctly solves the problem and explains his/her reasoning.
1	Student shows good understanding of the problem but does not fully solve or explain his/her reasoning.
0	Student does not demonstrate understanding of the problem.

H.O.T. Focus on Higher Order Thinking

18. Critical Thinking An equation has the form $\frac{a}{x} + \frac{x}{b} = c$, where a, b, and c are constants and $b \neq 0$. How many solutions could this equation have? Explain. **See margin.**

19. Multiple Representations Write an equation whose graph is a straight line, but with an open circle at $x = 4$. **See margin.**

20. Justify Reasoning Explain why the excluded values do not change when multiplying by the LCD to add or subtract rational expressions. **The function was defined not to exist at those points, so if an alternate form of the equation is to be equivalent, the new form must not exist at those points either.**

21. Critical Thinking Describe how you would find the inverse of the rational function $f(x) = \frac{x-1}{x-2}$, $x \neq 2$. Then find the inverse. **See Additional Answers.**

Lesson Performance Task

Kasey creates comedy sketch videos and posts them on a popular video website and is selling an exclusive series of sketches on DVD. The total cost to make the series of sketches is \$989. The materials cost \$1.40 per DVD and the shipping costs \$2.00 per DVD. Kasey plans to sell the DVDs for \$12 each.

a. Let d be the number of DVDs Kasey sells. Create a profit-per-item model from the given information by writing a rule for $C(d)$, the total costs in dollars, $S(d)$, the total sales income in dollars, $P(d)$, the profit in dollars, and $P_{PI}(d)$, the profit per item sold in dollars.

b. What is the profit per DVD if Kasey sells 80 DVDs? Does this value make sense in the context of the problem?

c. Then use the function $P_{PI}(d)$ from part a to find how many DVDs Kasey would have to sell to break even. Identify all excluded values.

a. $C(d) = 989 + 1.4d + 2d = 989 + 3.4d$

$S(d) = 12d$

$P(d) = 12d - (989 + 3.4d) = 8.6d - 989$

$P_{PI}(d) = \frac{8.6d - 989}{d}$

b. $P_{PI}(80) = \frac{8.6(80) - 989}{(80)}$

$= \frac{-301}{80}$

≈ -3.76

Kasey would have a profit of −\$3.76 per DVD sold. This value makes sense because the costs are greater than the amount of sales.

c. The excluded value is $d = 0$.

$0 = \frac{8.6d - 989}{d}$

$0 \cdot d = \frac{8.6d - 989}{d} \cdot d$

$0 = 8.6d - 989$

$989 = 8.6d$

$115 = d$

Kasey would have to sell 115 DVDs to break even.

Extension Activity

Have students consider starting a bakery business to sell bread at a farmer's market. Have students choose a price per loaf and estimate the number of loaves they would have to sell in order to make a profit, assuming they start by buying a 50-pound bag of flour. Then have students create a model for the profit per loaf of bread, taking into account all relevant costs. After inputting all known values, have students discuss whether they would adjust the price or the sales goal in order to make a profit.