9.2 Multiplying and Dividing Rational Expressions

Essential Question: How can you multiply and divide rational expressions?

Resource Locker

Explore Relating Multiplication Concepts

Use the facts you know about multiplying rational numbers to determine how to multiply rational expressions.

(A) How do you multiply $\frac{4}{5} \cdot \frac{5}{6}$?

Multiply 4 by ? **5** to find the ? **numerator** of the product, and multiply 5 by ? **6** to find the ? **denominator**.

(B) $\frac{4}{5} \cdot \frac{5}{6} = \frac{?}{?}$ **20** / **30**

(C) To simplify, factor the numerator and denominator.

$20 =$? **2 · 2 · 5**

$30 =$? **2 · 3 · 5**

(D) Cancel common factors in the numerator and denominator to simplify the product.

$\frac{4}{5} \cdot \frac{5}{6} = \frac{20}{30} = \frac{2 \cdot 2 \cdot 5}{2 \cdot 3 \cdot 5} = \frac{?}{?}$ **2** / **3**

(E) Based on the steps used for multiplying rational numbers, how can you multiply the rational expression $\frac{x+1}{x-1} \cdot \frac{3}{2(x+1)}$? ? **Multiply $x + 1$ by 3 to find the numerator of the product, and multiply $x - 1$ by $2(x + 1)$ to find the denominator. Then cancel common factors to simplify the product.**

Reflect

1. **Discussion** Multiplying rational expressions is similar to multiplying rational numbers. Likewise, dividing rational expressions is similar to dividing rational numbers. How could you use the steps for dividing rational numbers to divide rational expressions?
When dividing rational numbers, multiply by the reciprocal of the divisor and follow the steps for multiplying rational numbers. So, when dividing rational expressions, multiply by the reciprocal of the divisor and follow the steps for multiplying rational expressions.

Multiplying and Dividing Rational Expressions

Learning Objective

Students will multiply and divide rational expressions, solve problems using multiplication and division of rational expressions, and explain the closure properties of rational expressions.

Math Processes and Practices

MPP8 Generalizing

Language Objective

Explain to a partner the steps for multiplying and dividing rational expressions.

Online Resources

An extra example for each Explain section is available online.

Engage

Essential Question: How can you multiply and divide rational expressions?

To find the product of rational expressions, factor each numerator and denominator, multiply the numerators and denominators, and simplify the resulting rational expression that is the product. To find the quotient of rational expressions, multiply the dividend by the reciprocal of the divisor and then follow the steps for multiplying rational expressions.

Preview: Lesson Performance Task

View the Engage section online. Discuss the photo and how the heat generated by a runner's body could depend on height. Then preview the Lesson Performance Task.

Professional Development

Learning Progressions

Students learned how to simplify rational expressions in the previous lesson. They also know how to multiply and divide numerical fractions. Here, they combine those skills to multiply and divide rational expressions. Students apply their knowledge of factoring, as well as of multiplying polynomials, to simplify expressions involving multiplication and division of rational expressions. The concept of excluded values will carry over into later studies, for example, in excluding extraneous values in the simplification of logarithms.

Explore

Relating Multiplication Concepts

Integrate Technology

Students have the option of completing the Explore activity either in the book or online.

Questioning Strategies

What are two different ways of multiplying $\frac{2x^2y}{6xy} \cdot \frac{3y}{4x}$? Multiply across and then simplify the result, or divide out common factors of the numerators and denominators and then multiply across. In either case, the result will be $\frac{y}{4}$.

Explain 1

Multiplying Rational Expressions

Avoid Common Errors

Students sometimes confuse multiplying rational expressions with cross-multiplying. Point out that cross-multiplying takes place across an equal sign when solving equations of the form $\frac{a}{b} = \frac{c}{d}$. Tell students to use the equal sign as the cue to cross multiply. When multiplying rational expressions, multiply straight across.

Questioning Strategies

Why should you factor the numerators and the denominators before you multiply? It makes it easier to multiply because you can divide out common factors from a numerator and a denominator before multiplying.

Explain 1 Multiplying Rational Expressions

To multiply rational expressions, multiply the numerators to find the numerator of the product, and multiply the denominators to find the denominator. Then, simplify the product by cancelling common factors.

Note the excluded values of the product, which are any values of the variable for which the expression is undefined.

Example 1 **Find the products and any excluded values.**

(A) $\frac{3x^2}{x^2-2x-8} \cdot \frac{2x^2-6x-20}{x^2-3x-10}$

$$\frac{3x^2}{x^2-2x-8} \cdot \frac{2x^2-6x-20}{x^2-3x-10} = \frac{3x^2}{(x+2)(x-4)} \cdot \frac{2(x+2)(x-5)}{(x+2)(x-5)}$$

Factor the numerators and denominators.

$$= \frac{6x^2(x+2)(x-5)}{(x+2)(x-4)(x+2)(x-5)}$$

Multiply the numerators and multiply the denominators.

$$= \frac{6x^2\cancel{(x+2)}\cancel{(x-5)}}{\cancel{(x+2)}(x-4)(x+2)\cancel{(x-5)}}$$

Cancel the common factors in the numerator and denominator.

$$= \frac{6x^2}{(x+2)(x-4)}$$

Determine what values of x make each expression undefined.

$\frac{3x^2}{x^2-2x-8}$: The denominator is 0 when $x = -2$ and $x = 4$.

$\frac{2x^2-6x-20}{x^2-3x-10}$: The denominator is 0 when $x = -2$ and $x = 5$.

Excluded values: $x = -2$, $x = 4$, and $x = 5$

(B) $\frac{x^2-8x}{14(x^2+8x+15)} \cdot \frac{7x+35}{x+8}$

$$\frac{x^2-8x}{14(x^2+8x+15)} \cdot \frac{7x+35}{x+8} = \frac{\boxed{x}\,(x-8)}{14\left(\boxed{x+3}\right)(x+5)} \cdot \frac{7\left(\boxed{x+5}\right)}{x+8}$$

Factor the numerators and denominators.

$$= \frac{7x(x-8)\left(\boxed{x+5}\right)}{14\left(\boxed{x+3}\right)(x+5)(x+8)}$$

Multiply the numerators and multiply the denominators.

$$= \frac{\boxed{x(x-8)}}{\boxed{2(x+3)(x+8)}}$$

Cancel the common factors in the numerator and denominator.

Determine what values of x make each expression undefined.

$\frac{x^2-8x}{14(x^2+8x+15)}$: The denominator is 0 when $\boxed{x = -3 \text{ and } x = -5}$.

$\frac{7x+35}{x+8}$: The denominator is 0 when $\boxed{x = -8}$.

Excluded values: $\boxed{x = -3,\ x = -5, \text{ and } x = -8}$

Collaborative Learning

Peer-to-Peer Activity

Have students work in pairs. Instruct each pair to create a problem involving the division of two rational expressions by working backward from the factored form of the numerators and denominators. Have them rewrite the problem, multiplying the factors in each numerator and denominator. Then have them exchange problems with another pair, and find the quotient. Have each pair compare their answer to the answer determined by the students who created the problem.

Your Turn

Find the products and any excluded values.

2. $\dfrac{x^2-9}{x^2-5x-24} \cdot \dfrac{x-8}{2x^2-18x}$ $\dfrac{(x-3)}{2x(x-9)}$

Excluded values: $x=-3$, $x=8$, $x=0$, and $x=9$

3. $\dfrac{x}{x-9} \cdot \dfrac{3x-27}{x+1}$ $\dfrac{3x}{x+1}$

Excluded values: $x=-1$ and $x=9$

Explain 2 Dividing Rational Expressions

To divide rational expressions, change the division problem to a multiplication problem by multiplying by the reciprocal. Then, follow the steps for multiplying rational expressions.

Example 2 Find the quotients and any excluded values.

Ⓐ $\dfrac{(x+7)^2}{x^2} \div \dfrac{x^2+9x+14}{x^2+x-2}$

$$\frac{(x+7)^2}{x^2} \div \frac{x^2+9x+14}{x^2+x-2} = \frac{(x+7)^2}{x^2} \cdot \frac{x^2+x-2}{x^2+9x+14}$$ Multiply by the reciprocal.

$$= \frac{(x+7)(x+7)}{x^2} \cdot \frac{(x+2)(x-1)}{(x+7)(x+2)}$$ Factor the numerators and denominators.

$$= \frac{(x+7)(x+7)(x+2)(x-1)}{x^2(x+7)(x+2)}$$ Multiply the numerators and multiply the denominators.

$$= \frac{\cancel{(x+7)}(x+7)\cancel{(x+2)}(x-1)}{x^2\cancel{(x+7)}\cancel{(x+2)}}$$ Cancel the common factors in the numerator and denominator.

$$= \frac{(x+7)(x-1)}{x^2}$$

Determine what values of x make each expression undefined.

$\dfrac{(x+7)^2}{x}$: The denominator is 0 when $x=0$.

$\dfrac{x^2+9x+14}{x^2+x-2}$: The denominator is 0 when $x=-2$ and $x=1$.

$\dfrac{x^2+x-2}{x^2+9x+14}$: The denominator is 0 when $x=-7$ and $x=-2$.

Excluded values: $x=0$, $x=-7$, $x=1$, and $x=-2$

Ⓑ $\dfrac{6x}{3x-30} \div \dfrac{9x^2-27x-36}{x^2-10x}$

$$\frac{6x}{3x-30} \div \frac{9x^2-27x-36}{x^2-10x} = \frac{6x}{3x-30} \cdot \frac{\boxed{x^2-10x}}{\boxed{9x^2-27x-36}}$$ Multiply by the reciprocal.

$$= \frac{6x}{3\left(\boxed{x-10}\right)} \cdot \frac{x\left(\boxed{x-10}\right)}{9(x+1)\left(\boxed{x-4}\right)}$$ Factor the numerators and denominators.

Differentiate Instruction

Graphic Organizers

Have students copy and complete the graphic organizer shown below, writing a worked-out example in each box.

	Numerical Fractions	Rational Expressions
Adding		
Subtracting		
Multiplying		
Dividing		

Integrate Math Processes and Practices

Focus on Abstract and Quantitative Reasoning

MPP2 Students should recognize that multiplying two rational expressions does not introduce excluded values. The excluded values of the product are the combined excluded values of the original rational expressions. Students can use this fact to help detect errors in their work.

Explain 2

Dividing Rational Expressions

Questioning Strategies

How is the procedure for dividing rational expressions related to multiplying rational expressions? Dividing by an expression is equivalent to multiplying by its reciprocal. Once division is converted to multiplication, you can carry out the steps for multiplying rational expressions.

Why must you exclude values of the variable that make the numerator of the divisor 0? If the numerator of a fraction is 0, then the fraction equals 0. Since division by 0 is undefined, the divisor cannot be equal to 0.

Integrate Math Processes and Practices

Focus on Using and Evaluating Logical Reasoning

MPP3 Prompt students to recognize that they can check their solutions to division problems by multiplying the quotient by the divisor and checking to see that the result is the dividend.

Explain 3

Activity: Investigating Closure

Avoid Common Errors

Students may think that a single example is sufficient to prove that a set is closed. While a single counterexample is enough to prove that a set is not closed, the general result must be proven to show closure. For example, the quotient of the integer division $8 \div 2 = 4$ is an integer, but the integers are not closed under division.

Questioning Strategies

How do you determine whether a set of polynomials or rational expressions is closed under a given operation? Define the members of the set. Then investigate the set to determine whether the given operation always results in a member of the set.

Integrate Math Processes and Practices

Focus on Abstract and Quantitative Reasoning

MPP2 For most students, it will be easier to give a counter example to show that a set is *not* closed than to explain why a set *is* closed. Encourage students to use variables such as *a* and *b* to represent elements of the set, and try to determine the general result of the operation on *a* and *b*.

$$= \frac{6x^2\left(\boxed{x-10}\right)}{27\left(\boxed{x-10}\right)(x+1)\left(\boxed{x-4}\right)}$$

Multiply the numerators and multiply the denominators.

$$= \frac{\boxed{2x^2}}{\boxed{9(x+1)(x-4)}}$$

Cancel the common factors in the numerator and denominator.

Determine what values of x make each expression undefined.

$\frac{6x}{3x-30}$: The denominator is 0 when $\boxed{x = 10}$.

$\frac{9x^2-27x-36}{x^2-10x}$: The denominator is 0 when $\boxed{x = 10 \text{ and } x = 0}$.

$\frac{x^2-10x}{9x^2-27x-36}$: The denominator is 0 when $\boxed{x = -1 \text{ and } x = 4}$.

Excluded values: $\boxed{x = 0,\ x = 10,\ x = -1, \text{ and } x = 4}$

Your Turn

Find the quotients and any excluded values.

4. $\frac{x+11}{4x} \div \frac{2x+6}{x^2+2x-3}$ $\frac{(x+11)(x-1)}{8x}$

Excluded values: $x = 0$, $x = 1$, and $x = -3$

5. $\frac{20}{x^2-7x} \div \frac{5x^2-40x}{x^2-15x+56}$ $\frac{4}{x^2}$

Excluded values: $x = 0$, $x = 7$, and $x = 8$

Explain 3 Activity: Investigating Closure

A set of numbers is said to be closed, or to have **closure**, under a given operation if the result of the operation on any two numbers in the set is also in the set.

A Recall whether the set of whole numbers, the set of integers, and the set of rational numbers are closed under each of the four basic operations.

	Addition	Subtraction	Multiplication	Division
Whole Numbers	Closed ?	Not Closed ?	Closed ?	Not Closed ?
Integers	Closed ?	Closed ?	Closed ?	Not Closed ?
Rational Numbers	Closed ?	Closed ?	Closed ?	Closed ?

B Look at the set of rational expressions. Use the rational expressions $\frac{p(x)}{q(x)}$ and $\frac{r(x)}{s(x)}$ where $p(x)$, $q(x)$, $r(x)$ and $s(x)$ are nonzero. Add the rational expressions.

$$\frac{p(x)}{q(x)} + \frac{r(x)}{s(x)} = \boxed{?}\ \frac{p(x)s(x) + q(x)r(x)}{q(x)s(x)}$$

C Is the set of rational expressions closed under addition? Explain. $\boxed{?}$

Yes; since $q(x)$ and $s(x)$ are nonzero, $q(x)s(x)$ is nonzero. So, $\frac{p(x)s(x) + q(x)r(x)}{q(x)s(x)}$ is again a rational expression.

Language Support EL

Communicate Math

Have students work in pairs. Provide each pair of students with some rational expressions to multiply or divide, written on sticky notes or index cards. Have the first student explain the steps to multiply rational expressions while the second student writes notes. Students switch roles and repeat the process for a division problem, highlighting the additional step of using the reciprocal.

(D) Subtract the rational expressions.

$$\frac{p(x)}{q(x)} - \frac{r(x)}{s(x)} = \boxed{?} \quad \frac{p(x)s(x) - q(x)r(x)}{q(x)s(x)}$$

(E) Is the set of rational expressions closed under subtraction? Explain. $\boxed{?}$ **Yes; since $q(x)$ and $s(x)$ are nonzero, $q(x)s(x)$ is nonzero. So, $\frac{p(x)s(x) - q(x)r(x)}{q(x)s(x)}$ is again a rational expression.**

(F) Multiply the rational expressions.

$$\frac{p(x)}{q(x)} \cdot \frac{r(x)}{s(x)} = \boxed{?} \quad \frac{p(x)r(x)}{q(x)s(x)}$$

(G) Is the set of rational expressions closed under multiplication? Explain. $\boxed{?}$ **Yes; since $q(x)$ and $s(x)$ are nonzero, $q(x)s(x)$ is nonzero. So, $\frac{p(x)r(x)}{q(x)s(x)}$ is again a rational expression.**

(H) Divide the rational expressions.

$$\frac{p(x)}{q(x)} \div \frac{r(x)}{s(x)} = \boxed{?} \quad \frac{p(x)s(x)}{q(x)r(x)}$$

(I) Is the set of rational expressions closed under division? Explain. $\boxed{?}$ **Yes; since $q(x)$ and $r(x)$ are nonzero, $q(x)r(x)$ is nonzero. So, $\frac{p(x)s(x)}{q(x)r(x)}$ is again a rational expression.**

Reflect

6. Are rational expressions most like whole numbers, integers, or rational numbers? Explain.

6. Rational expressions are like rational numbers because both the set of rational expressions and the set of rational numbers are closed under all four basic operations.

Explain 4 Multiplying and Dividing with Rational Models

Models involving rational expressions can be solved using the same steps to multiply or divide rational expressions.

Example 3 **Solve the problems using rational expressions.**

(A) Leonard drives 40 miles to work every day. One-fifth of his drive is on city roads, where he averages 30 miles per hour. The other part of his drive is on a highway, where he averages 55 miles per hour. The expression $\frac{d_c r_h + d_h r_c}{r_c r_h}$ represents the total time spent driving, in hours. In the expression, d_c represents the distance traveled on city roads, d_h represents the distance traveled on the highway, r_c is the average speed on city roads, and r_h is the average speed on the highway. Use the expression to find the average speed of Leonard's drive.

The total distance traveled is 40 miles. Find an expression for the average speed, r, of Leonard's drive.

$$r = \text{Total distance traveled} \div \text{Total time}$$
$$= 40 \div \frac{d_c r_h + d_h r_c}{r_c r_h}$$
$$= 40 \cdot \frac{r_c r_h}{d_c r_h + d_h r_c}$$
$$= \frac{40 r_c r_h}{d_c r_h + d_h r_c}$$

Explain 4

Multiplying and Dividing with Rational Models

Questioning Strategies

How do you determine the excluded values in a real-world problem that involves dividing two rational expressions? Find the values that make each denominator 0 and that make the numerator of the divisor 0. Also, determine numbers that are not reasonable values for the independent variable in the situation.

Integrate Math Processes and Practices Focus on Mathematical Modeling

MPP4 Discuss with students how the rational expressions used in the example model the situation. Discuss what each numerator and denominator represents, and why a quotient of these quantities is an appropriate model.

Elaborate

Integrate Math Processes and Practices

Focus on Using Precise Mathematical Language

MPP6 Call upon students to describe each step involved in the solution to a problem involving division of two rational expressions. Make sure they use accurate mathematical language in describing not only the division process, but also how to identify excluded values of the variable.

Summarize The Lesson

How do you divide two rational expressions? Multiply the first rational expression by the reciprocal of the second. Factor each numerator and denominator, and then multiply numerators and multiply denominators. Divide out common factors of the numerators and denominators.

Find the values of d_c and d_h.

$d_c = \frac{1}{5}(40) = 8$ miles

$d_h = 40 - 8 = 32$ miles

Solve for r by substituting in the given values from the problem.

$$r = \frac{dr_c r_h}{d_c r_h + d_h r_c} = \frac{40 \cdot 55 \cdot 30}{8 \cdot 55 + 32 \cdot 30} \approx 47 \text{ miles per hour}$$

The average speed of Leonard's drive is about 47 miles per hour.

B) The fuel efficiency of Tanika's car at highway speeds is 35 miles per gallon. The expression $\frac{48E - 216}{E(E-6)}$ represents the total gas consumed, in gallons, when Tanika drives 36 miles on a highway and 12 miles in a town to get to her relative's house. In the expression, E represents the fuel efficiency, in miles per gallon, of Tanika's car at highway speeds. Use the expression to find the average rate of gas consumed on her trip.

The total distance traveled is $\boxed{48}$ miles. Find an expression for the average rate of gas consumed, g, on Tanika's trip.

$g =$ Total gas consumed $\div$ Total distance traveled

$$= \frac{48E - 216}{E(E-6)} \div \boxed{48}$$

$$= \frac{48E - 216}{\boxed{48}\, E(E-6)}$$

The value of E is $\boxed{35}$.

Solve for g by substituting in the value of E.

$$g = \frac{48\left(\boxed{35}\right) - 216}{48\left(\boxed{35}\right)\left(\boxed{35} - 6\right)}$$

$$= \frac{\boxed{1464}}{\boxed{48{,}720}}$$

$$\approx \boxed{0.03}$$

The average rate of gas consumed on Tanika's trip is about $\boxed{0.03}$ gallon per mile.

Your Turn

7. The distance traveled by a car undergoing constant acceleration, a, for a time, t, is given by $d = v_0 t + \frac{1}{2}at^2$, where v_0 is the initial velocity of the car. Two cars are side by side with the same initial velocity. One car accelerates and the other car does not. Write an expression for the ratio of the distance traveled by the accelerating car to the distance traveled by the nonaccelerating car as a function of time.
The ratio as a function of time is $1 + \frac{At}{2v_0}$.

Elaborate

8. Explain how finding excluded values when dividing one rational expression by another is different from multiplying two rational expressions. **See margin.**

9. **Essential Question Check-In** How is dividing rational expressions related to multiplying rational expressions?
When dividing rational expressions, find the reciprocal of the divisor and change the division problem to a multiplication problem. Then follow the steps for multiplying rational expressions.

Answers

8. When finding excluded values of a product of two rational expressions, find the values of x for which the denominator of either expression is 0. When finding excluded values when dividing one rational expression by another, find the values of x for which the denominator of either expression or the numerator of the second expression is 0.

Evaluate: Homework and Practice

Personal Math Trainer

- Online Homework
- Hints and Help
- Extra Practice

1. Explain how to multiply the rational expressions.
$\frac{x-3}{2} \cdot \frac{x^2-3x+4}{x^2-2x}$
Multiply $x - 3$ by $x^2 - 3x + 4$ to get the numerator of the product. Multiply 2 by $x^2 - 2x$ to get the denominator of the product. Then, simplify by cancelling common factors in the numerator and the denominator.

Find the products and any excluded values.

2. $\frac{x}{3x-6} \cdot \frac{x-2}{x+9}$ **$\frac{x}{3(x+9)}$**
Excluded values: $x = 2$ and $x = -9$

3. $\frac{5x^2+25x}{2} \cdot \frac{4x}{x+5}$ **$10x^2$**
Excluded value: $x = -5$

4. $\frac{x^2-2x-15}{10x+30} \cdot \frac{3}{x^2-3x-10}$
See Additional Answers.

5. $\frac{x^2-1}{x^2+5x+4} \cdot \frac{x^2}{x^2-x}$
See Additional Answers.

6. $\frac{x^2+14x+33}{4x} \cdot \frac{x^2-3x}{x+3} \cdot \frac{8x-56}{x^2+4x-77}$
$2(x-3)$; Excluded values: $x = 0, x = -3, x = -11$, and $x = 7$

7. $\frac{9x^2}{x-6} \cdot \frac{x^2-36}{3x-6} \cdot \frac{3}{4x^2+24x}$
$\frac{9x}{4(x-2)}$; Excluded values: $x = 6, x = 2, x = 0$ and $x = -6$

Find the quotients and any excluded values.

8. $\frac{5x^2+10x}{x^2+2x+1} \div \frac{20x+40}{x^2-1}$ **$\frac{x(x-1)}{4(x+1)}$**
Excluded values: $x = 1, x = -1$, and $x = -2$

9. $\frac{x^2-9x+18}{x^2+9x+18} \div \frac{x^2-36}{x^2-9}$ **$\frac{(x-3)^2}{(x+6)^2}$**
Excluded values: $x = \pm 6, x = \pm 3$

10. $\frac{-x^2+x+20}{5x^2-25x} \div \frac{x+4}{2x-14}$ **$\frac{-2(x-7)}{5x}$**
Excluded values: $x = 0, x = 5, x = 7$ and $x = -4$

11. $\frac{x+3}{x^2+8x+15} \div \frac{x^2-25}{x-5}$ **$\frac{1}{(x+5)^2}$**
Excluded values: $x = -5, x = -3$, and $x = 5$

12. $\frac{x^2-10x+9}{3x} \div \frac{x^2-7x-18}{x^2+2x}$ **$\frac{x-1}{3}$**
Excluded values: $x = 0, x = -2$, and $x = 9$

13. $\frac{8x+32}{x^2+8x+16} \div \frac{x^2-6x}{x^2-2x-24}$ **$\frac{8}{x}$**
Excluded values: $x = 0, x = -4$, and $x = 6$

Let $p(x) = \frac{1}{x+1}$ and $q(x) = \frac{1}{x-1}$. Perform the operation, and show that it results in another rational expression.

14. $p(x) + q(x)$ **$\frac{2x}{(x+1)(x-1)}$; the numerator and denominator are polynomials, so it is a rational expression.**

15. $p(x) - q(x)$ **$\frac{-2}{(x+1)(x-1)}$; the numerator and denominator are polynomials, so it is a rational expression.**

16. $p(x) \cdot q(x)$
See Additional Answers.

17. $p(x) \div q(x)$ **$\frac{x-1}{x+1}$; the numerator and denominator are polynomials, so it is a rational expression.**

18. The distance a race car travels is given by the equation $d = v_0 t + \frac{1}{2}at^2$, where v_0 is the initial speed of the race car, a is the acceleration, and t is the time travelled. Near the beginning of a race, the driver accelerates for 9 seconds at a rate of 4 m/s^2. The driver's initial speed was 75 m/s. Find the driver's average speed during the acceleration.
The average speed during the acceleration is 93 meters per second.

19. Julianna is designing a circular track that will consist of three concentric rings. The radius of the middle ring is 6 meters greater than that of the inner ring and 6 meters less than that of the outer ring. Find an expression for the ratio of the length of the outer ring to the length of the middle ring and another for the ratio of the length of the outer ring to length of the inner ring. If the radius of the inner ring is set at 90 meters, how many times longer is the outer ring than the middle ring and the inner ring?
See Additional Answers.

Exercise	Depth of Knowledge (D.O.K.)	Math Processes and Practices
1	**1** Recall of Information	**MPP6** Using Precise Mathematical Language
2–13	**1** Recall of Information	**MPP2** Abstract and Quantitative Reasoning
14–17	**2** Skills/Concepts	**MPP6** Using Precise Mathematical Language
18–20	**2** Skills/Concepts	**MPP4** Mathematical Modeling
21–22	**3** Strategic Thinking H.O.T.	**MPP2** Abstract and Quantitative Reasoning
23	**2** Skills/Concepts H.O.T.	**MPP4** Mathematical Modeling

Evaluate

Personal Math Trainer

Assignment Guide

Level	Concepts and Skills	Practice
Basic	Explore	Exercise 1
	Example 1	Exercises 2–4
	Example 2	Exercises 8–10
	Example 3	Exercise 18
	H.O.T.	Exercise 23
Average	Explore	Exercise 1
	Example 1	Exercises 3–5
	Example 2	Exercises 8–10
	Example 3	Exercises 18–19
	H.O.T.	Exercise 21, 23
Advanced	Explore	N/A
	Example 1	Exercises 6–7
	Example 2	Exercises 11–13
	Example 3	Exercises 18–20
	H.O.T.	Exercises 21–23

Real World Problems

Avoid Common Errors

When multiplying rational expressions, students may divide out by common factors and then, erroneously, cross-multiply instead of multiplying straight across. Remind them that cross-multiplying is used to solve equations, and that when multiplying two rational expressions, they must multiply straight across.

Avoid Common Errors

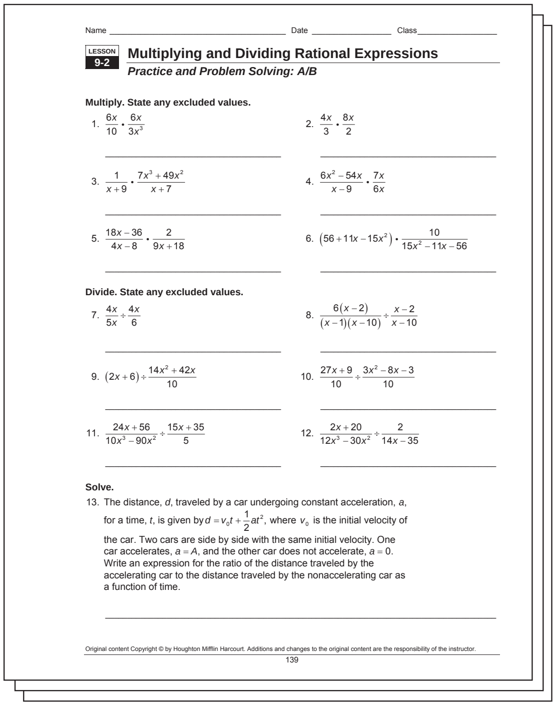
When identifying excluded values for quotients of rational expressions, students may consider values that cause the denominators to be zero, but they may forget to consider values that cause the divisor itself to be 0. For example, the divisor $\frac{x^2-36}{x^2-4x}$ will have a value of 0 when $x = 6$ or $x = -6$, so these values must also be excluded values.

Journal

Have students compare and contrast the method they have learned for multiplying rational expressions with the method they have learned for adding rational expressions.

Online Resources

- Practice and Problem Solving (three forms)
- Reteach
- Reading Strategies
- Success for English Learners


Name ______________________ Date ______________ Class ______________

LESSON 9-2 **Multiplying and Dividing Rational Expressions**
Practice and Problem Solving: A/B

Multiply. State any excluded values.

1. $\frac{6x}{10} \cdot \frac{6x}{3x^3}$
2. $\frac{4x}{3} \cdot \frac{8x}{2}$
3. $\frac{1}{x+9} \cdot \frac{7x^3+49x^2}{x+7}$
4. $\frac{6x^2-54x}{x-9} \cdot \frac{7x}{6x}$
5. $\frac{18x-36}{4x-8} \cdot \frac{2}{9x+18}$
6. $(56+11x-15x^2) \cdot \frac{10}{15x^2-11x-56}$

Divide. State any excluded values.

7. $\frac{4x}{5x} \div \frac{4x}{6}$
8. $\frac{6(x-2)}{(x-1)(x-10)} \div \frac{x-2}{x-10}$
9. $(2x+6) \div \frac{14x^2+42x}{10}$
10. $\frac{27x+9}{10} \div \frac{3x^2-8x-3}{10}$
11. $\frac{24x+56}{10x^3-90x^2} \div \frac{15x+35}{5}$
12. $\frac{2x+20}{12x^3-30x^2} \div \frac{2}{14x-35}$

Solve.

13. The distance, d, traveled by a car undergoing constant acceleration, a, for a time, t, is given by $d = v_0t + \frac{1}{2}at^2$, where v_0 is the initial velocity of the car. Two cars are side by side with the same initial velocity. One car accelerates, $a = A$, and the other car does not accelerate, $a = 0$. Write an expression for the ratio of the distance traveled by the accelerating car to the distance traveled by the nonaccelerating car as a function of time.

Answers

21. Maria did not multiply by the reciprocal.

$$\frac{x^2-4x-45}{3x-15} \div \frac{6x^2-150}{x^2-5x} = \frac{x^2-4x-45}{3x-15} \cdot \frac{x^2-5x}{6x^2-150}$$
$$= \frac{(x-9)(x+5)}{3(x-5)} \cdot \frac{x(x-5)}{6(x+5)(x-5)}$$
$$= \frac{x(x-9)(x+5)(x-5)}{18x(x-5)(x+5)(x-5)}$$
$$= \frac{x(x-9)}{18(x-5)}$$

Lesson Performance Task Scoring Rubric

Points	Criteria
2	Student correctly solves the problem and explains his/her reasoning.
1	Student shows good understanding of the problem but does not fully solve or explain his/her reasoning.
0	Student does not demonstrate understanding of the problem.

20. **Geometry** Find a rational expression for the ratio of the surface area of a cylinder to the volume of a cylinder. Then find the ratio when the radius is 3 inches and the height is 10 inches. $\frac{2(r+h)}{rh}$

The ratio of the cylinder's surface area to its volume is 13:15.

H.O.T. Focus on Higher Order Thinking

21. **Explain the Error** Maria finds an expression equivalent to $\frac{x^2-4x-45}{3x-15} \div \frac{6x^2-150}{x^2-5x}$. Her work is shown. Find and correct Maria's mistake. **See margin.**

$$\frac{x^2-4x-45}{3x-15} \div \frac{6x^2-150}{x^2-5x} = \frac{(x-9)(x+5)}{3(x-5)} \div \frac{6(x+5)(x-5)}{x(x-5)}$$
$$= \frac{6(x-9)(x+5)(x+5)(x-5)}{3x(x-5)(x-5)}$$
$$= \frac{2(x-9)(x+5)^2}{x(x-5)}$$

22. **Critical Thinking** Multiply the expressions. What do you notice about the resulting expression?

$$\left(\frac{3}{x-4} + \frac{x^3-4x}{8x^2-32}\right)\left(\frac{3x+18}{x^2+2x-24} - \frac{x}{8}\right)$$

$\frac{9}{(x-4)^2} - \frac{x^2}{64}$

The expression is the difference of two squares.

23. **Multi-Step** Jordan is making a garden with an area of $x^2 + 13x + 30$ square feet and a length of $x + 3$ feet. **See Additional Answers.**

a. Find an expression for the width of Jordan's garden.

b. If Karl makes a garden with an area of $3x^2 + 48x + 180$ square feet and a length of $x + 6$, how many times larger is the width of Jon's garden than Jordan's?

c. If x is equal to 4, what are the dimensions of both Jordan's and Karl's gardens?

Lesson Performance Task

Who has the advantage, taller or shorter runners? Almost all of the energy generated by a long-distance runner is released in the form of heat. For a runner with height H and speed V, the rate h_g of heat generated and the rate h_r of heat released can be modeled by $h_g = k_1H^3V^2$ and $h_r = k_2H^2$, k_1 and k_2 being constants. So, how does a runner's height affect the amount of heat she releases as she increases her speed?

First, set up the ratio for the amount of heat generated by the runner to the amount of heat released by dividing the value h_g by h_r: $\frac{h_g}{h_r} = \frac{k_1H^3V^2}{k_2H^2}$

Next, simplify the ratio: $\frac{h_g}{h_r} = \frac{k_1HV^2}{k_2}$

When $\frac{h_g}{h_r}$ is equal to 1, the amount of heat released is the same as the amount of heat generated. You can use this condition as a way to determine the relationship of height to speed. Setting the ratio equal to 1, isolate speed on one side of the equation: $\frac{k_2}{k_1H} = V^2$

Since k_1 and k_2 are constants, you see that as a runner's height increases, the speed required to maintain the balance of heat generated to heat released gets smaller. Therefore, a shorter runner can run at a higher speed and not lose as much heat as a taller runner does.

Extension Activity

Ask students to rework the problem, this time with the heat generated modeled by $h_g = k_1H^3V$. Ask them to describe the relation between speed and height, and to tell how that relation differs from the answer they calculated in the Performance Task. Ask them whether this model gives shorter runners a greater or lesser advantage, compared to the model in the Performance Task. Shorter runners still have an advantage over taller runners, but it is not as great.