9.1 Adding and Subtracting Rational Expressions

Essential Question: How can you add and subtract rational expressions?

Resource Locker

Explore Identifying Excluded Values

Given a rational expression, identify the excluded values by finding the zeroes of the denominator. If possible, simplify the expression.

Ⓐ $\dfrac{(1-x^2)}{x-1}$

The denominator of the expression is ?. $x-1$

Ⓑ Since division by 0 is not defined, the excluded values for this expression are all the values that would make the denominator equal to 0.

$x - 1 = 0$

$x = ?$ **1**

Ⓒ Begin simplifying the expression by factoring the numerator.

$\dfrac{(1-x^2)}{x-1} = \dfrac{(?)(?)}{x-1}$ $1-x$; $1+x$

Ⓓ Divide out terms common to both the numerator and the denominator.

$\dfrac{1-x^2}{x-1} = \dfrac{(?)(?)}{-1} - x = ? = ?$ $1-x$; $1+x$; $-(1+x)$; $-1-x$

Ⓔ The simplified expression is

$\dfrac{(1-x^2)}{x-1} = ?$, whenever $x \neq ?$ $-1-x$; **1**

Ⓕ What is the domain for this function? What is its range? ?

Its domain is all real numbers except $x = 1$. Its range is all real numbers except $y = -2$.

Reflect

1. What factors can be divided out of the numerator and denominator?

Common factors

Professional Development

Integrate Math Processes and Practices

Operations on rational expressions are similar to operations on fractions. For example,

$$\frac{a(x)}{b(x)} + \frac{c(x)}{d(x)} = \frac{a(x)d(x) + b(x)c(x)}{b(x)d(x)}$$

except where $b(x) = 0$ and/or $d(x) = 0$. As with fractions, it is generally best to simplify rational expressions before adding, subtracting, multiplying, or dividing. Note that in the above equation, the denominator $b(x)d(x)$ may not be the *least* common denominator of the two rational expressions.

Adding and Subtracting Rational Expressions

Learning Objective

Students will write equivalent rational expressions by using factors, find the Least Common Denominator of two rational expressions, and add and subtract rational expressions.

Math Processes and Practices

MPP7 Seeing Structure

Language Objective

Explain to a partner how to simplify a rational expression and how to add and subtract rational expressions.

Online Resources

An extra example for each Explain section is available online.

Engage

Essential Question: How can you add and subtract rational expressions?

Possible answer: Convert them to like denominators, then add the numerators, all the while keeping track of the combined excluded values.

Preview: Lesson Performance Task

View the Engage section online. Discuss the photo and how the river's current can either help the kayaker go faster or slow the kayaker down. Then preview the Lesson Performance Task.

Explore

Identifying Excluded Values

Integrate Technology

Students have the option of completing the Explore activity either in the book or online.

Questioning Strategies

When identifying excluded values, why is it not necessary to consider values of the variable that make the numerator equal to 0? If the numerator is 0 and the denominator is nonzero, the fraction is defined and is equal to 0, so values that make the numerator 0 do not need to be excluded (as long as they don't also make the denominator 0).

Why are there no excluded values for the rational expression $\frac{x}{x^2+4}$? There are no real numbers for which $x^2 + 4$ equals 0.

Explain 1

Writing Equivalent Rational Expressions

Avoid Common Errors

Students sometimes make errors writing the simplified form of a rational expression in which all factors of the numerator divide out with common factors from the denominator. These students may write the simplified denominator as the final answer, forgetting that there is a factor of 1 that remains in the numerator. To help students avoid this error, encourage them to write a 1 above or below any factor that divides out, and to multiply the 1's with any remaining factors when writing the final rational expression.

Explain 1 Writing Equivalent Rational Expressions

Given a rational expression, there are different ways to write an equivalent rational expression. When common terms are divided out, the result is an equivalent but simplified expression.

Example 1 **Rewrite the expression as indicated.**

(A) Write $\frac{3x}{(x+3)}$ as an equivalent rational expression that has a denominator of $(x+3)(x+5)$.

The expression $\frac{3x}{(x+3)}$ has a denominator of $(x+3)$.

The factor missing from the denominator is $(x+5)$.

Introduce a common factor, $(x+5)$.

$$\frac{3x}{(x+3)} = \frac{3x(x+5)}{(x+3)(x+5)}$$

$\frac{3x}{(x+3)}$ is equivalent to $\frac{3x(x+5)}{(x+3)(x+5)}$.

(B) Simplify the expression $\frac{(x^2+5x+6)}{(x^2+3x+2)(x+3)}$.

Write the expression.	$\frac{(x^2+5x+6)}{(x^2+3x+2)(x+3)}$
Factor the numerator and denominator.	$\frac{(x+2)(x+3)}{(x+1)(x+2)(x+3)}$
Divide out like terms.	$\frac{1}{x+1}$

Your Turn

2. Write $\frac{5}{5x-25}$ as an equivalent expression with a denominator of $(x-5)(x+1)$. $\frac{x+1}{(x-5)(x+1)}$

3. Simplify the expression $\frac{(x+x^3)(1-x^2)}{(x^2-x^6)}$. $\frac{1}{x}$

Explain 2 Identifying the LCD of Two Rational Expressions

Given two or more rational expressions, the least common denominator (LCD) is found by factoring each denominator and finding the least common multiple (LCM) of the factors. This technique is useful for the addition and subtraction of expressions with unlike denominators.

Least Common Denominator (LCD) of Rational Expressions

To find the LCD of rational expressions:

1. Factor each denominator completely. Write any repeated factors as powers.
2. List the different factors. If the denominators have common factors, use the highest power of each common factor.

Collaborative Learning

Peer-to-Peer Activity

Pair advanced students with classmates who are struggling with adding and subtracting rational expressions with unlike denominators. Encourage the advanced students to start by trying to diagnose which part of the process is causing the most trouble and to focus on helping with the appropriate skills. By explaining the process, advanced students will develop a deeper understanding of the concepts and struggling students will benefit from peer instruction.

Example 2 Find the LCD for each set of rational expressions.

(A) $\frac{-2}{3x-15}$ and $\frac{6x}{4x+28}$

Factor each denominator completely.

$3x - 15 = 3(x - 5)$

$4x + 28 = 4(x + 7)$

List the different factors.

$3, 4, x - 5, x + 7$

The LCD is $3 \cdot 4(x-5)(x+7)$,

or $12(x-5)(x+7)$.

(B) $\frac{-14}{x^2-11x+24}$ and $\frac{9}{x^2-6x+9}$

Factor each denominator completely.

$x^2 - 11x + 24 = \boxed{(x-3)(x-8)}$

$x^2 - 6x + 9 = \boxed{(x-3)(x-3)}$

List the different factors. $x - 3$ and $x - 8$

Taking the highest power of $(x - 3)$, the LCD is $(x-3)^2(x-8)$.

Reflect

4. **Discussion** When is the LCD of two rational expressions not equal to the product of their denominators?

When a factor appears one or more times in each denominator

Your Turn

Find the LCD for each set of rational expressions.

5. $\frac{x+6}{8x-24}$ and $\frac{14x}{10x-30}$

$\mathbf{2^3 \cdot 5(x-3) = 40(x-3)}$

6. $\frac{12x}{15x+60}$ and $\frac{5}{x^2+9x+20}$

$\mathbf{3 \cdot 5(x+4)(x+5) = 15(x+4)(x+5)}$

Explain 3 Adding and Subtracting Rational Expressions

Adding and subtracting rational expressions is similar to adding and subtracting fractions.

Example 3 Add or subtract. Identify any excluded values and simplify your answer.

(A) $\frac{x^2+4x+2}{x^2} + \frac{x^2}{x^2+x}$

Factor the denominators. $\frac{x^2+4x+2}{x^2} + \frac{x^2}{x(x+1)}$

Identify where the expression is not defined. The first expression is undefined when $x = 0$. The second expression is undefined when $x = 0$ and when $x = -1$.

Find a common denominator. The LCM for x^2 and $x(x+1)$ is $x^2(x+1)$.

Write the expressions with a common denominator by multiplying both by the appropriate form of 1.

$$\frac{(x+1)}{(x+1)} \cdot \frac{x^2+4x+2}{x^2} + \frac{x^2}{x(x+1)} \cdot \frac{x}{x}$$

Simplify each numerator.

$$= \frac{x^3+5x^2+6x+2}{x^2(x+1)} + \frac{x^3}{x^2(x+1)}$$

Add.

$$= \frac{2x^3+5x^2+6x+2}{x^2(x+1)}$$

Since none of the factors of the denominator are factors of the numerator, the expression cannot be further simplified.

Questioning Strategies

When simplifying a rational expression, when might it be helpful to factor −1 from either the numerator or the denominator? when the expressions in the numerator and the denominator are not both written in descending form, or when the leading coefficient of one or both expressions is negative

Integrate Math Processes and Practices

Focus on Problem Solving

MPP1 To help students understand why rational expressions such as $\frac{x+2}{2}$ cannot be simplified, have them substitute a number, such as 6, for x and compare the value of $\frac{6+2}{2}$ to $\frac{6+\cancel{2}^{1}}{\cancel{2}_{1}}$. Point out that the two expressions are not equivalent. Tell students they can use this substitute-and-check strategy in other situations when they are not sure whether their action produces an equivalent expression.

Explain 3

Adding and Subtracting Rational Expressions

Avoid Common Errors

Students may make sign errors when combining the numerators in a problem involving the subtraction of two rational expressions. To avoid this, reinforce the importance of writing the numerator being subtracted in parentheses and carefully applying the distributive property before combining like terms.

Differentiate Instruction

Communicating Math

It may be beneficial to have students verbally describe the steps involved in adding and subtracting rational expressions. Ensure that students' descriptions are accurate, but allow them to use their own words to describe the steps. Pay careful attention, in students' explanations, to how they find the common denominator, and how they convert the numerators of the fractions to obtain equivalent expressions. Students may also benefit from hearing how other students describe these steps.

Questioning Strategies

How do you find the LCD of rational expressions? First, factor each denominator. Then write the product of the factors of the denominators. If the denominators have common factors, use the highest power of that factor found in any of the denominators.

What steps do you use to rewrite the expressions with like denominators? Multiply each numerator by any factors of the LCD that were not factors of its original denominator. Use the LCD for the denominator.

Peer-to-Peer Discussion

Ask students to consider whether the product of the denominators can always be used as the common denominator when adding or subtracting fractions with unlike denominators. Have them experiment to see what happens in situations in which the product is not the *least* common denominator. The product can be used, but the result will need to be simplified.

Explain 4

Adding and Subtracting with Rational Models

Questioning Strategies

What is the significance of the excluded values of a rational expression that models a real-world situation? The excluded values are numbers that are not possible values of the independent variable in the given situation.

(B) $\dfrac{2x^2}{x^2-5x} - \dfrac{x^2+3x-4}{x^2}$

Factor the denominators. $\quad \dfrac{2x^2}{\boxed{x(x-5)}} - \dfrac{x^2+3x-4}{x^2}$

Identify where the expression is not defined. The first expression is undefined when $x = 0$ and when $x = 5$. The second expression is undefined when $x = 0$.

Find a common denominator. The LCM for $x(x-5)$ and x^2 is $x^2(x-5)$.

Write the expressions with a common denominator by multiplying both by the appropriate form of 1. $\quad \boxed{\dfrac{x}{x}} \cdot \dfrac{2x^2}{x(x-5)} - \dfrac{x^2+3x-4}{x^2} \cdot \dfrac{x-5}{x-5}$

Simplify each numerator. $\quad = \dfrac{2x^3}{x^2(x-5)} - \dfrac{x^3-2x^2-19x+20}{x^2(x-5)}$

Subtract. $\quad = \dfrac{\boxed{x^3} + 2x^2 + 19x - 20}{x^2(x-5)}$

Since none of the factors of the denominator are factors of the numerator, the expression cannot be further simplified.

Your Turn

Add each pair of expressions, simplifying the result and noting the combined excluded values. Then subtract the second expression from the first, again simplifying the result and noting the combined excluded values.

7. $-x^2$ and $\dfrac{1}{(1-x^2)}$ **See below.**

8. $\dfrac{x^2}{(4-x^2)}$ and $\dfrac{1}{(2-x)}$ **See below.**

Explain 4 Adding and Subtracting with Rational Models

Rational expressions can model real-world phenomena, and can be used to calculate measurements of those phenomena.

Example 4 **Find the sum or difference of the models to solve the problem.**

(A) Two groups have agreed that each will contribute \$2000 for an upcoming trip. Group A has 6 more people than group B. Let x represent the number of people in group A. Write and simplify an expression in terms of x that represents the difference between the number of dollars each person in group A must contribute and the number each person in group B must contribute.

$$\frac{2000}{x} - \frac{2000}{x-6} = \frac{2000(x-6)}{x(x-6)} - \frac{2000x}{(x-6)x}$$
$$= \frac{2000x - 12{,}000 - 2000x}{x(x-6)}$$
$$= -\frac{12{,}000}{x(x-6)}$$

7. Addition: $\dfrac{x^4-x^2+1}{(1+x)(1-x)}, x \neq \pm 1$

Subtraction: $\dfrac{x^4-x^2-1}{(1+x)(1-x)}, x \neq \pm 1$

8. Addition: $\dfrac{x^2+x+2}{(2+x)(2-x)}, x \neq \pm 2$

Subtraction: $\dfrac{-(x+1)}{(2+x)}, x \neq \pm 2$

Language Support EL

Communicate Math

Have students work in pairs. Provide each pair of students with some rational expressions written on sticky notes or index cards and with some addition and subtraction problems. Have the first student explain the steps to simplify a rational expression while the second student writes notes. Students switch roles and repeat the process with an addition problem, then again with a subtraction problem.

(B) A freight train averages 30 miles per hour traveling to its destination with full cars and 40 miles per hour on the return trip with empty cars. Find the total time in terms of d. Use the formula $t = \frac{d}{r}$.

Let d represent the one-way distance.

$$\text{Total time: } \frac{d}{30} + \frac{d}{40} = \frac{d \cdot \boxed{40}}{30 \cdot \boxed{40}} + \frac{d \cdot \boxed{30}}{40 \cdot \boxed{30}}$$

$$= \frac{d \cdot \boxed{40} + d \cdot \boxed{30}}{\boxed{1200}}$$

$$= \frac{\boxed{7}}{\boxed{120}} d$$

Your Turn

9. A hiker averages 1.4 miles per hour when walking downhill on a mountain trail and 0.8 miles per hour on the return trip when walking uphill. Find the total time in terms of d. Use the formula $t = \frac{d}{r}$.

Let d represent the one-way distance.

Total time: $\frac{d}{1.4} + \frac{d}{0.8} = \frac{55}{28}d$

10. Yvette ran at an average speed of 6.20 feet per second during the first two laps of a race and an average speed of 7.75 feet per second during the second two laps of a race. Find her total time in terms of d, the distance around the racecourse. **Total time: $\frac{(2)(13.95)d}{48.05} \approx 0.58d$**

Elaborate

11. Why do rational expressions have excluded values? **See below.**

12. How can you tell if your answer is written in simplest form? **See below.**

13. **Essential Question Check-In** Why must the excluded values of each expression in a sum or difference of rational expressions also be excluded values for the simplified expression?
You cannot add or subtract undefined expressions so any value that makes one expression in a sum or difference undefined has to be an excluded value for the simplified expression.

11. Excluded values would make the expression undefined.

12. When none of the factors of the denominator are factors of the numerator, the answer is written in simplest form.

Integrate Math Processes and Practices
Focus on Using Mathematical Tools

MPP5 Students can use a graphing calculator to compare the graph of the function defined by the original sum or difference with the graph of the function defined by the final simplified expression. The graphs should be identical.

Elaborate

Integrate Math Processes and Practices
Focus on Using and Evaluating Logical Reasoning

MPP3 Ask students to discuss why values that are excluded from an addend in a sum of rational expressions must be excluded from the final simplified form of the sum, even if that value does not make the simplified expression undefined.

Questioning Strategies

Why does the rational expression $\frac{x^2+1}{x^2-1}$ have two excluded values, but the expression $\frac{x^2-1}{x^2+1}$ have none? The denominator of the first rational expression is equal to 0 when $x = 1$ and when $x = -1$. There is no real number that makes the denominator of the second expression equal to 0.

Summarize The Lesson

How do you subtract two rational expressions? If the denominators are like, subtract the numerators and write the result as the numerator over the common denominator. If the denominators are not alike, find the LCD, convert each rational expression to an equivalent expression having the LCD, and then follow the steps described above. If the second numerator contains more than one term, be careful to apply the distributive property when subtracting.

Evaluate

Personal Math Trainer

Assignment Guide

Level	Concepts and Skills	Practice
Basic	Explore	Exercises 1–2
	Example 1	Exercises 3–6
	Example 2	Exercises 9–11
	Example 3	Exercises 15–18
	Example 4	Exercise 25
	H.O.T.	Exercise 32
Average	Explore	Exercises 1–2
	Example 1	Exercises 4–7
	Example 2	Exercises 10–12
	Example 3	Exercises 19–22
	Example 4	Exercises 25–26
	H.O.T.	Exercises 30–31
Advanced	Explore	Exercise 1
	Example 1	Exercises 6–8
	Example 2	Exercises 12–14
	Example 3	Exercises 21–24
	Example 4	Exercises 26–28
	H.O.T.	Exercises 30–32

Real World Problems

Questioning Strategies

Why do some rational expressions have excluded values while others do not? The denominators of some rational expressions contain factors that are equal to 0 for one or more values of x. Those rational expressions will have excluded values, since the rational expression is undefined for those values of x. In other rational expressions, no value of x makes the denominator equal to 0, so there are no excluded values.

Avoid Common Errors

Students often list only the values that make the simplest form of a rational expression undefined. Stress the importance of recording all of the values for which the original expression is undefined.

Evaluate: Homework and Practice

Personal Math Trainer

- Online Homework
- Hints and Help
- Extra Practice

Given a rational expression, identify the excluded values by finding the zeroes of the denominator.

1. $\dfrac{x-1}{x^2+3x-4}$ $\quad \dfrac{x-1}{(x+4)(x-1)}$ $x \neq 1, x \neq -4$

2. $\dfrac{4}{x(x+17)}$ $\quad \dfrac{4}{x(x+17)}$ $x \neq 0, x \neq -17$

Write the given expression as an equivalent rational expression that has the given denominator.

3. Expression: $\dfrac{x-7}{x+8}$ Denominator: x^3+8x^2 $\quad \dfrac{x^2(x-7)}{x^3+8x^2}$ or $\dfrac{x^3-7x^2}{x^3+8x^2}$

4. Expression: $\dfrac{3x^3}{3x-6}$ Denominator: $(2-x)(x^2+9)$ $\quad \dfrac{-x^3(x^2+9)}{(2-x)(x^2+9)}$

Simplify the given expression.

5. $\dfrac{(-4-4x)}{(x^2-x-2)}$ $\quad \dfrac{4}{(2-x)}$

6. $\dfrac{-x-8}{x^2+9x+8}$ $\quad \dfrac{-1}{(x+1)}$

7. $\dfrac{6x^2+5x+1}{3x^2+4x+1}$ $\quad \dfrac{(2x+1)}{(x+1)}$

8. $\dfrac{x^4-1}{x^2+1}$ $\quad (x^2-1)$

Find the LCD for each set of rational expressions.

9. $\dfrac{x}{2x+16}$ and $\dfrac{-4x}{3x-27}$ $\quad 6(x+8)(x-9)$

10. $\dfrac{x^2-4}{5x-30}$ and $\dfrac{5x+13}{7x-42}$ $\quad 35(x-6)$

11. $\dfrac{4x+12}{x^2+5x+6}$ and $\dfrac{5x+15}{10x+20}$ $\quad 10(x+2)(x+3)$

12. $\dfrac{-11}{x^2-3x-28}$ and $\dfrac{2}{x^2-2x-24}$ See margin.

13. $\dfrac{12}{3x^2-21x-54}$ and $\dfrac{-1}{21x^2-84}$ See margin.

14. $\dfrac{3x}{5x^2-40x+60}$ and $\dfrac{17}{-7x^2+56x-84}$ See margin.

Add or subtract the given expressions, simplifying each result and noting the combined excluded values.

15. $\dfrac{1}{1+x}+\dfrac{1-x}{x}$ $\quad \dfrac{-x^2+x+1}{x(1+x)}, x \neq 0, -1$

16. $\dfrac{x+4}{x^2-4}+\dfrac{-2x-2}{x^2-4}$ $\quad \dfrac{-1}{(x+2)}, x \neq \pm 2$

17. $\dfrac{1}{2+x}-\dfrac{2-x}{x}$ $\quad \dfrac{x^2+x-4}{x(x+2)}, x \neq 0, -2$

18. $\dfrac{4x^4+4}{x^2+1}-\dfrac{8}{x^2+1}$ $\quad 4(x^2-1)$ $(x^2+1)=0$ has no real solutions so no values are excluded.

19. $\dfrac{x^4-2}{x^2-2}+\dfrac{2}{-x^2+2}$ $\quad x^2+2, x \neq \pm\sqrt{2}$

20. $\dfrac{1}{x^2+3x-4}-\dfrac{1}{x^2-3x+2}$ See margin.

21. $\dfrac{3}{x^2-4}-\dfrac{x+5}{x+2}$ $\quad \dfrac{-x^2-3x+13}{(x+2)(x-2)}, x \neq \pm 2$

22. $\dfrac{-3}{9x^2-4}+\dfrac{1}{3x^2+2x}$ $\quad \dfrac{-2}{x(3x+2)(3x-2)}, x \neq 0, \pm\frac{2}{3}$

23. $\dfrac{x-2}{x+2}+\dfrac{1}{x^2-4}-\dfrac{x+2}{2-x}$ $\quad \dfrac{2x^2+9}{(x+2)(x-2)}, x \neq \pm 2$

24. $\dfrac{x-3}{x+3}-\dfrac{1}{x-3}+\dfrac{x+2}{3-x}$ $\quad \dfrac{-12x}{(x-3)(x+3)}, x \neq \pm 3$

25. A company has two factories, factory A and factory B. The cost per item to produce q items in factory A is $\frac{200+13q}{q}$. The cost per item to produce q items in factory B is $\frac{300+25q}{2q}$. Find an expression for the sum of these costs per item. Then divide this expression by 2 to find an expression for the average cost per item to produce q items in each factory. See margin.

Exercise	Depth of Knowledge (D.O.K.)	Math Processes and Practices
1–2	1 Recall of Information	MPP5 Using Mathematical Tools
3–24	2 Skills/Concepts	MPP2 Abstract and Quantitative Reasoning
25–27	2 Skills/Concepts	MPP4 Mathematical Modeling
28	3 Strategic Thinking H.O.T.	MPP4 Mathematical Modeling
29–30	2 Skills/Concepts	MPP4 Mathematical Modeling
31–32	3 Strategic Thinking H.O.T.	MPP3 Using and Evaluating Logical Reasoning

26. An auto race consists of 8 laps. A driver completes the first 3 laps at an average speed of 185 miles per hour and the remaining laps at an average speed of 200 miles per hour. Let d represent the length of one lap. Find the time in terms of d that it takes the driver to complete the race.

$$\frac{3d}{185} + \frac{5d}{200} = \frac{3d(200)}{185(200)} + \frac{5d(185)}{200(185)}$$
$$= \frac{600d + 925d}{37{,}000}$$
$$= \frac{61d}{1480} \text{ hours}$$

27. The junior and senior classes of a high school are cleaning up a beach. Each class has pledged to clean 1600 meters of shoreline. The junior class has 12 more students than the senior class. Let s represent the number of students in the senior class. Write and simplify an expression in terms of s that represents the difference between the number of meters of shoreline each senior must clean and the number of meters each junior must clean. **See margin.**

28. Architecture The Renaissance architect Andrea Palladio believed that the height of a room with vaulted ceilings should be the harmonic mean of the length and width. The harmonic mean of two positive numbers a and b is equal to $\frac{2}{\frac{1}{a}+\frac{1}{b}}$. Simplify this expression. What are the excluded values? What do they mean in this problem? **See margin.**

29. Match each expression with the correct excluded value(s).

A. $\dfrac{3x+5}{x+2}$ **C** a. ___?___ no excluded values

B. $\dfrac{1+x}{x^2-1}$ **D** b. ___?___ $x \neq 0, -2$

C. $\dfrac{3x^4-12}{x^2+4}$ **B** c. ___?___ $x \neq 1, -1$

D. $\dfrac{3x+6}{x^2(x+2)}$ **A** d. ___?___ $x \neq -2$

H.O.T. Focus on Higher Order Thinking

30. Explain the Error George was asked to write the expression $2x - 3$ three times, once each with excluded values at $x = 1$, $x = 2$, and $x = -3$. He wrote the following expressions:

a. $\dfrac{2x-3}{x-1}$

b. $\dfrac{2x-3}{x-2}$

c. $\dfrac{2x-3}{x+3}$

George wrote expressions that had the correct excluded values, but his expressions are not equivalent to $2x - 3$. The correct expressions should be the following:

a. $\dfrac{(2x-3)(x-1)}{x-1}$

b. $\dfrac{(2x-3)(x-2)}{x-2}$

c. $\dfrac{(2x-3)(x+3)}{x+3}$

$2x - 3 = \dfrac{(2x-3)(x-1)(x-2)(x+3)}{(x-1)(x-2)(x+3)}, x \neq 1, 2, -3$

What error did George make? Write the correct expressions, then write an expression that has all three excluded values.

31. Communicate Mathematical Ideas Write a rational expression with excluded values at $x = 0$ and $x = 17$. **Answers may vary. Sample answer:** $\dfrac{a}{x(x-17)}$

Modeling

When working with rational expressions that represent real-world situations, students should recognize that not only must they consider excluded values that are based on the algebraic nature of the denominators of the expressions, but they also need to consider values that must be excluded due to the limitations on the domain in the given situation.

Visual Cues

When students simplify rational expressions, suggest that they highlight each different factor in the numerator and denominator with a different color by using highlighters or colored pencils. This process can help them keep track of common factors, as shown below.

$$\frac{x^2(x+1)(x-2)^2}{x(x+3)(x+1)} = \frac{x(x-2)^2}{x+3}$$

Connect Vocabulary EL

Have students explain why a rational expression has excluded values for a denominator of zero, using what they know about fractions or ratios to explain. Help them by re–voicing or clarifying their explanations, as needed.

Journal

Have students compare the method used to add two rational numbers to the method used to add two rational expressions. Have them use specific examples to illustrate their explanations.

Answers

12. $(x-7)(x-6)(x+4)$

13. $21(x-9)(x+2)(x-2)$

14. $-35(x-6)(x-2)$

20. $= \dfrac{-6}{(x-1)(x+4)(x-2)}, x \neq -4, 1, 2$

25. $\dfrac{700+51q}{2q}$

So the average cost per item to produce q items in each factory is $\dfrac{700+51q}{4q}$.

Answers

27. $$\frac{1600}{s} - \frac{1600}{s+12} = \frac{1600(s+12)}{s(s+12)} - \frac{1600s}{(s+12)s}$$
$$= \frac{1600s + 19{,}200 - 1600s}{s(s+12)}$$
$$= \frac{19{,}200}{s(s+12)} \text{ meters}$$

28. $$\frac{2}{\frac{1}{a}+\frac{1}{b}} = \frac{2}{\frac{1b}{ab}+\frac{1a}{ba}} = \frac{2}{\left(\frac{b+a}{ab}\right)} = \frac{2ab}{a+b}, \text{ excluding } a = 0, b = 0, a = -b.$$

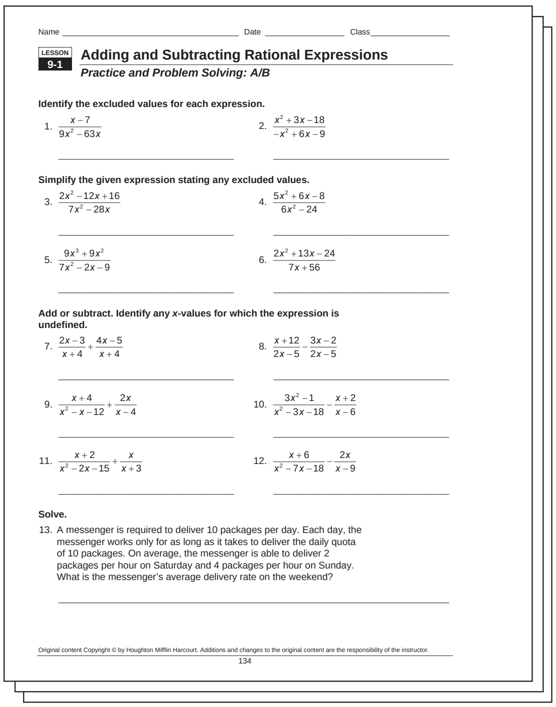
These values do not occur because geometric lengths are positive.

Avoid Common Errors

Students may divide the total distance (6 miles) by the total time (4 hours) to get an average speed of 1.5 miles per hour, but this is incorrect. Ask students why the upstream and downstream travel times need to be calculated separately. The kayaker is traveling at a different speed in each direction due to the river current. Ask students what quantity is the same in both directions. distance Ask students what quantities are different in each direction. the travel time and speed of the kayaker

Online Resources

- Practice and Problem Solving (three forms)
- Reteach
- Reading Strategies
- Success for English Learners



Name ______________ Date __________ Class __________

LESSON 9-1 **Adding and Subtracting Rational Expressions**

Practice and Problem Solving: A/B

Identify the excluded values for each expression.

1. $\frac{x-7}{9x^2-63x}$
2. $\frac{x^2+3x-18}{-x^2+6x-9}$

Simplify the given expression stating any excluded values.

3. $\frac{2x^2-12x+16}{7x^2-28x}$
4. $\frac{5x^2+6x-8}{6x^2-24}$
5. $\frac{9x^3+9x^2}{7x^2-2x-9}$
6. $\frac{2x^2+13x-24}{7x+56}$

Add or subtract. Identify any *x*-values for which the expression is undefined.

7. $\frac{2x-3}{x+4}+\frac{4x-5}{x+4}$
8. $\frac{x+12}{2x-5}-\frac{3x-2}{2x-5}$
9. $\frac{x+4}{x^2-x-12}+\frac{2x}{x-4}$
10. $\frac{3x^2-1}{x^2-3x-18}-\frac{x+2}{x-6}$
11. $\frac{x+2}{x^2-2x-15}+\frac{x}{x+3}$
12. $\frac{x+6}{x^2-7x-18}-\frac{2x}{x-9}$

Solve.

13. A messenger is required to deliver 10 packages per day. Each day, the messenger works only for as long as it takes to deliver the daily quota of 10 packages. On average, the messenger is able to deliver 2 packages per hour on Saturday and 4 packages per hour on Sunday. What is the messenger's average delivery rate on the weekend?

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Lesson Performance Task Scoring Rubric

Points	Criteria
2	Student correctly solves the problem and explains his/her reasoning.
1	Student shows good understanding of the problem but does not fully solve or explain his/her reasoning.
0	Student does not demonstrate understanding of the problem.

32. Critical Thinking Sketch the graph of the rational equation $y = \frac{x^2+3x+2}{x+1}$. Think about how to show graphically that a graph exists over a domain except at one point. **See below.**

Lesson Performance Task

A kayaker spends an afternoon paddling on a river. She travels 3 miles upstream and 3 miles downstream in a total of 4 hours. In still water, the kayaker can travel at an average speed of 2 miles per hour. Based on this information, can you estimate the average speed of the river's current? Is your answer reasonable?

Next, assume the average speed of the kayaker is an unknown, k, and not necessarily 2 miles per hour. What is the range of possible average kayaker speeds under the rest of the constraints?

Total time is equal to time upstream plus time downstream.

$$T = \frac{D}{r_1} + \frac{D}{r_2} = D\left(\frac{1}{r_1} + \frac{1}{r_2}\right) = D\left(\frac{r_1 + r_2}{r_1 r_2}\right)$$

$$\frac{T}{D}(r_1 r_2) = r_1 + r_2$$

Substitute for r_1 and r_2:

$$\frac{T}{D}(2-c)(2+c) = (2-c) + (2+c)$$

$$\frac{T}{D}(4-c^2) = 4$$

$$(4-c^2) = 4\frac{D}{T}$$

$$c^2 - 4 = -4\frac{D}{T}$$

Plug in specific constants:

$$c^2 = -4\frac{D}{T} + 4 = -4\left(\frac{3}{4}\right) + 4 = -3 + 4 = 1$$

$$c = \pm\sqrt{1} = \pm 1$$

The answer is 1 mile per hour only because the speed in this context cannot be negative.
To figure out the range of speeds of the kayaker in the river, we substitute k for 2 above.

$$\frac{T}{D}(k-c)(k+c) = (k-c) + (k+c) = 2k$$

$$(k^2 - c^2) = 2k\frac{D}{T}$$

$$c^2 - k^2 = -2k\frac{D}{T}$$

$$c^2 = -2k\frac{D}{T} + k^2$$

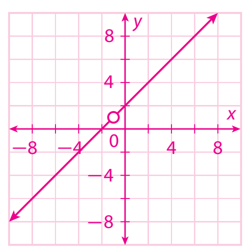
$$c = \pm\sqrt{-2k\frac{D}{T} + k^2} \rightarrow -2k\frac{D}{T} + k^2 \geq 0$$

$$k^2 \geq 2k\frac{D}{T}$$

$$k \geq 2\frac{D}{T} = 2 \cdot \frac{3}{4} = 1.5 \text{ miles per hour}$$

The kayaker has to go at least 1.5 miles per hour regardless of the speed of the current.

32.



This expression has an excluded value at $x = -1$. This can be shown on the graph with a hole at that one point.

Extension Activity

Have students explain how the presence of a strong wind might affect the time it takes the kayaker to travel up and down the river. Ask the students how they would account for the wind in their calculations. Add or subtract a term w for the effect of the wind on the kayaker's speed. For example, if the wind's effect is acting in the same direction as the current, the kayaker's speed upstream would be $r_1 = 2 - c - w$ and downstream would be $r_2 = 2 + c + w$. It the wind's effect is acting in the opposite direction of the current, the kayaker's speed upstream would be $r_1 = 2 - c + w$ and downstream would be $r_2 = 2 + c - w$.