1.4 Inverses of Functions

Essential Question: What is an inverse function, and how do you know it's an inverse function?

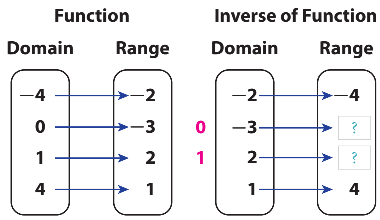
Resource Locker

Explore Understanding Inverses of Functions

Recall that a *relation* is any pairing of the elements of one set (the domain) with the elements of a second set (the range). The elements of the domain are called inputs, while the elements of the range are called outputs. A function is a special type of relation that pairs every input with exactly one output. In a *one-to-one function*, no output is ever used more than once in the function's pairings. In a *many-to-one function*, at least one output is used more than once.

An **inverse relation** reverses the pairings of a relation. If a relation pairs an input x with an output y, then the inverse relation pairs an input y with an output x. The inverse of a function may or may not be another function. If the inverse of a function $f(x)$ is also a function, it is called the **inverse function** and is written $f^{-1}(x)$. If the inverse of a function is not a function, then it is simply an inverse relation.

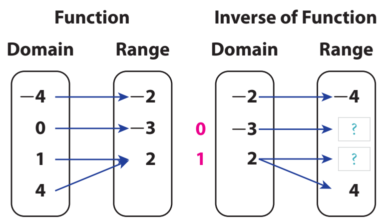
(A) The mapping diagrams show a function and its inverse. Complete the diagram for the inverse of the function.



Is the function one-to-one or many-to-one? Explain.

Is the inverse of the function also a function? Explain. **The function is one-to-one, because no output is ever used more than once in the function's pairings.**
The inverse is a function, because for each input there is only one output.

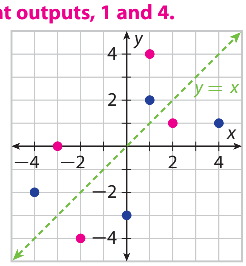
(B) The mapping diagrams show a function and its inverse. Complete the diagram for the inverse of the function.



Is the function one-to-one or many-to-one? Explain.

Is the inverse of the function also a function? Explain. **The function is many-to-one, because there are two inputs, 1 and 4, that have the same output.**
The inverse is not a function, because the input 2 has two different outputs, 1 and 4.

(C) The graph of the original function in Step A is shown. Note that the graph also shows the dashed line $y = x$. Write the inverse of the function as a set of ordered pairs and graph them along with the original function.



Function: $\{(-4, -2), (0, -3), (1, 2), (4, 1)\}$

Inverse of function: **$\{(-2, -4), (-3, 0), (2, 1), (1, 4)\}$**

$\{(?, ?), (?, ?), (?, ?), (?, ?)\}$

What do you observe about the graphs of the function and its inverse in relationship to the line $y = x$? Why does this make sense? **See margin.**

Inverses of Functions

Learning Objective

Students will use mapping diagrams, graphs, and composition of functions to understand inverses of functions, and find the inverse of a linear function.

Math Processes and Practices

MPP6 Using Precise Mathematical Language

Language Objective

Show or explain what the inverse of the graph of a function would look like and justify your reasoning to a partner.

Online Resources

An extra example for each Explain section is available online.

Engage

Essential Question: What is an inverse function, and how do you know it's an inverse function?

Possible answer: Given a function $f(x)$ that pairs domain values with range values, the inverse function $f^{-1}(x)$, if it exists, reverses those pairings. (The inverse function exists only when the original function is one-to-one or has had its domain restricted so that it becomes one-to-one.) The composition of $f(x)$ and $f^{-1}(x)$ is just x, and the graphs of $f(x)$ and $f^{-1}(x)$ are reflections across the line $y = x$.

Preview: Lesson Performance Task

View the Engage section online. Discuss the photo, asking which would be more helpful and why: to obtain femur length as a function of height, or use the inverse function to find height as a function of femur length? Then preview the Lesson Performance Task.

Answer to Explore C is on page 36.

Professional Development

Math Background

A relation is a mapping of the elements of one set of numbers to the elements of another set, which produces a set of ordered pairs. A relation is a function if, for every input, there is exactly one output. The graph of a relation is a function if it passes the vertical line test, that is, if every possible vertical line drawn through the graph intersects it at no more than one point.

A function takes an input, applies a rule, and produces an output. The inverse of that function will use that output as its input and apply a rule to give the input of the original function as its output.

Explore

Understanding Inverses of Functions

Integrate Technology

Students have the option of completing the Explore activity either in the book or online.

Avoid Common Errors

Students often read $f^{-1}(x)$ as raising a function to the −1 power. Stress that in this notation −1 is not an exponent, even though it is written as a superscript.

Questioning Strategies

What must be true about a function if its inverse is not a function? The function must pair at least two inputs with the same output.

Answer to Explore C

C. The graphs are reflections across the line $y = x$, which makes sense because the ordered pairs in a function and its inverse have their x- and y-coordinates reversed.

Answers

1. The range of the original relation is the domain of the inverse relation, and the range of the inverse relation is the domain of the original relation.
2. It will also equal x. The function $f^{-1}(x)$ maps the output of $f(x)$ back to its input, so $f\left(f^{-1}(x)\right)$ will then map that input back to the input of $f^{-1}(x)$.

(D) The **composition of two functions** $f(x)$ and $g(x)$, written $f\left(g(x)\right)$ and read as "f of g of x," is a new function that uses the output of $g(x)$ as the input of $f(x)$. For example, consider the functions f and g with the following rules.

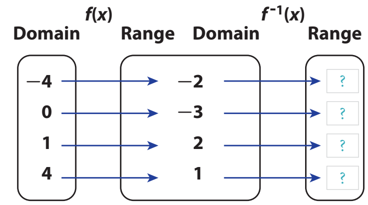
f: Add 1 to an input. g: Double an input.

Notice that $g(1) = 2(1) = 2$. So, $f\left(g(1)\right) = f(2) = 2 + 1 = 3$.

You can also find $g\left(f(x)\right)$. Notice that $f(1) = 1 + 1 = 2$. So, $g\left(f(1)\right) = g(2) = 2(2) = 4$.

For these two functions, you can see that $f\left(g(1)\right) \neq g\left(f(1)\right)$.

You can compose a function and its inverse. For instance, the mapping diagram shown illustrates $f^{-1}\left(f(x)\right)$ where $f(x)$ is the original function from Step A and $f^{-1}(x)$ is its inverse. Notice that the range of $f(x)$ serves as the domain of $f^{-1}(x)$. Complete the diagram. What do you notice about the outputs of $f^{-1}\left(f(x)\right)$? Explain why this makes sense. **See below.**



−4
0
1
4

Reflect **1–2. See margin.**

1. What is the relationship between the domain and range of a relation and its inverse?
2. **Discussion** In Step D, you saw that for inverse functions, $f^{-1}\left(f(x)\right) = x$. What do you expect $f\left(f^{-1}(x)\right)$ to equal? Explain.

Explain 1 Finding the Inverse of a Linear Function

Every linear function $f(x) = mx + b$ where $m \neq 0$ is a one-to-one function. So, its inverse is also a function. To find the inverse function, use the fact that inverse functions undo each other's pairings.

To find the inverse of a function $f(x)$:

1. Substitute y for $f(x)$.
2. Solve for x in terms of y.
3. Switch x and y (since the inverse switches inputs and outputs).
4. Replace y with $f^{-1}(x)$.

To check your work and verify that the functions are inverses, show that $f\left(f^{-1}(x)\right) = x$ and that $f^{-1}\left(f(x)\right) = x$.

D. The outputs of $f^{-1}\left(f(x)\right)$ exactly match the inputs of $f^{-1}\left(f(x)\right)$. That is, $f^{-1}\left(f(x)\right) = x$. The function takes each input and assigns it to an output. The inverse takes that output as an input, and assigns it back to the original function's input. So, the composition takes an input, assigns it to an output, and then assigns it back to the original input.

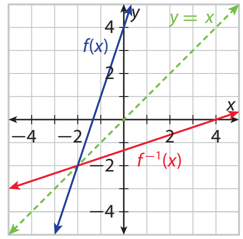
Collaborative Learning

Small Group Activity

Have students work in small groups. Have each group create a poster illustrating how inverse functions undo each other. Have them choose a function, and design an illustration that shows how the input and output values of the function and its inverse are related. Have groups share their posters with the class.

Example 1 **Find the inverse function $f^{-1}(x)$ for the given function $f(x)$. Use composition to verify that the functions are inverses. Then graph the function and its inverse.**

(A) $f(x) = 3x + 4$



Replace $f(x)$ with y. $\quad y = 3x + 4$

Solve for x. $\quad y - 4 = 3x$

$\frac{y-4}{3} = x$

Interchange x and y. $\quad y = \frac{x-4}{3}$

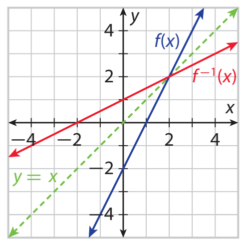
Replace y with $f^{-1}(x)$. $\quad f^{-1}(x) = \frac{x-4}{3}$

Check: Verify that $f^{-1}(f(x)) = x$ and $f(f^{-1}(x)) = x$.

$$f^{-1}(f(x)) = f^{-1}(3x+4) = \frac{(3x+4)-4}{3} = \frac{3x}{3} = x$$

$$f(f^{-1}(x)) = f\left(\frac{x-4}{3}\right) = 3\left(\frac{x-4}{3}\right) + 4 = (x-4) + 4 = x$$

(B) $f(x) = 2x - 2$



Replace $f(x)$ with y. $\quad y = \boxed{2x - 2}$

Solve for x. $\quad y \boxed{+2} = 2x$

$\frac{y+2}{2} = x$

Interchange x and y. $\quad y = \boxed{\frac{x+2}{2}}$

Replace y with $f^{-1}(x)$. $\quad \boxed{f^{-1}(x)} = \frac{x+2}{2}$

Check: Verify that $f^{-1}(f(x)) = x$ and $f(f^{-1}(x)) = x$.

$$f^{-1}(f(x)) = f^{-1}\left(\boxed{2x-2}\right) = \frac{(2x-2) + \boxed{2}}{\boxed{2}} = \frac{\boxed{2x}}{2} = \boxed{x}$$

$$f(f^{-1}(x)) = f\left(\boxed{\frac{x+2}{2}}\right) = \boxed{2}\left(\frac{x+2}{2}\right) - \boxed{2} = \left(\boxed{x+2}\right) - 2 = \boxed{x}$$

Reflect

3. The lines intersect on the line $y = x$, which is where the value of the function and its inverse are the same.

3. What is the significance of the point where the graph of a linear function and its inverse intersect?

4. The graph of a constant function $f(x) = c$ for any constant c is a horizontal line through the point $(0, c)$. Does a constant function have an inverse? Does it have an inverse function? Explain. **See margin.**

Your Turn

Find the inverse function $f^{-1}(x)$ for the given function $f(x)$. Use composition to verify that the functions are inverses. Then graph the function and its inverse.

5. $f(x) = -2x + 3$ **See margin.**

Explain 1

Finding the Inverse of a Linear Function

Questioning Strategies

A linear function takes each real number, multiplies it by 2 and subtracts 4. What does the inverse of this function do to its input values? The inverse function adds 4 to each input value, and divides the result by 2.

Integrate Math Processes and Practices Focus on Abstract and Quantitative Reasoning

MPP2 Encourage students always to test an inverse function using the fact that if $f(a) = b$, then $f^{-1}(b) = a$.

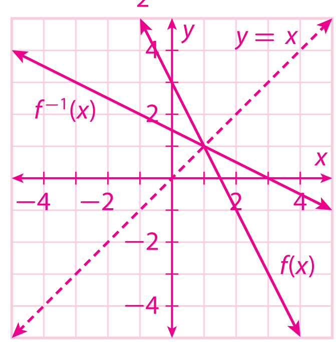
Questioning Strategies

If the graph of a linear function and its inverse function are parallel, what must be true about the slopes of the graphs of the two functions? Explain. The slope of each graph must be 1. These graphs will be parallel to the line $y = x$, whose slope is also 1.

Answers

4. A constant function has an inverse, but not an inverse function. Its inverse is the vertical line through $(c, 0)$. But because a constant function assigns every input to the same output, it is a many-to-one function, and its inverse is not a function.

5. $f^{-1}(x) = \frac{-x+3}{2}$



Differentiate Instruction

Visual Cues

Visual learners may benefit from folding their papers over the line $y = x$. This will enable them to see that $f(x)$ and $f^{-1}(x)$ are reflections of each other over that line.

Explain 2

Modeling with the Inverse of a Linear Function

Integrate Math Processes and Practices

Focus on Abstract and Quantitative Reasoning

MPP2 Inverses may also be thought of as a pair of relations that switch the role of the dependent and independent variables. The dependent variable becomes the independent variable, and vice versa.

Questioning Strategies

How can you use the graph of a linear function that models a real-world situation to write the rule for the inverse of the function? You can reflect the graph across the line $y = x$, identify the slope and the y-intercept of the image, and substitute into the equation $f(x) = mx + b$.

Integrate Technology

Students can use the table feature on a graphing calculator to observe that if (x, y) is a point on the graph of the original function, (y, x) is a point on the graph of the inverse function.

Connect Vocabulary EL

Show students examples of functions presented as input/output tables and as graphs, and have them explain why each function does or does not have an inverse function. Provide some terms and sentence stems that they can use to explain their reasoning, such as: "I know this function *(will/will not)* have an inverse because (*it is a one-to-one; it is a many to one; there is one x value for every y value*, etc.)." They should write their explanations using the terms and sentence stems, and then read their explanations aloud.

Explain 2 Modeling with the Inverse of a Linear Function

In a model for a real-world situation, the variables have specific real-world meanings. For example, the distance d (in miles) traveled in time t (in hours) at a constant speed of 60 miles per hour is $d = 60t$. Writing this in function notation as $d(t) = 60t$ emphasizes that this equation describes distance as a function of time.

You can find the inverse function for $d = 60t$ by solving for the independent variable t in terms of the dependent variable d. This gives the equation $t = \frac{d}{60}$. Writing this in function notation as $t(d) = \frac{d}{60}$ emphasizes that this equation describes time as a function of distance. Because the meanings of the variables can't be interchanged, you do not switch them at the end as you would switch x and y when working with purely mathematical functions. As you work with real-world models, you may have to restrict the domain and range.

Example 2 **For the given function, state the domain of the inverse function using set notation. Then find an equation for the inverse function, and graph it. Interpret the meaning of the inverse function.**

(A) The equation $C = 3.5g$ gives the cost C (in dollars) as a function of the number of gallons of gasoline g when the price is \$3.50 per gallon.

The domain of the function $C = 3.5g$ is restricted to nonnegative numbers to make real-world sense, so the range of the function also consists of nonnegative numbers. This means that the domain of the inverse function is $\{C \mid C \geq 0\}$.

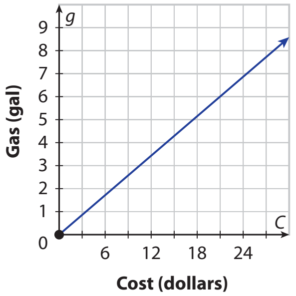
Solve the given equation for g to find the inverse function.

Write the equation. $C = 3.5g$

Divide both sides by 3.5. $\frac{C}{3.5} = g$

So, the inverse function is $g = \frac{C}{3.5}$.

Graph the inverse function.



The inverse function gives the number of gallons of gasoline as a function of the cost (in dollars) when the price of gas is \$3.50 per gallon.

Language Support EL

Communicate Math

Have students work in pairs. Instruct one student to choose a graph of a function and ask the partner to sketch what the inverse of the graph would look like. Once the partner completes the sketch, the first student asks, "How do you know?" They then switch roles and repeat the process with a different graph. Encourage the pairs to use terms such as *reflection, inverse, function,* $y = x$.

(B) A car's gas tank, which can hold 14 gallons of gas, contains 4 gallons of gas when the driver stops at a gas station to fill the tank. The gas pump dispenses gas at a rate of 5 gallons per minute. The equation $g = 5t + 4$ gives the number of gallons of gasoline g in the tank as a function of the pumping time t(in minutes).

The range of the function $g = 5t + 4$ is the number of gallons of gas in the tank, which varies from 4 gallons to 14 gallons. So, the domain of the inverse function is $\{g \mid 4 \leq g \leq 14\}$.

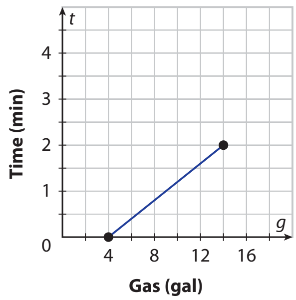
Solve the given equation for g to find the inverse function.

Write the equation. $g = 5t + 4$

Solve for t. $\frac{g-4}{5} = t$

So, the inverse function is $t = \frac{g-4}{5}$.

Graph the inverse function.



The inverse function gives the pumping time (in minutes) as a function of the amount of gas in the tank (in gallons) .

Your Turn

For the given function, determine the domain of the inverse function. Then find an equation for the inverse function, and graph it. Interpret the meaning of the inverse function.

6. A municipal swimming pool containing 600,000 gallons of water is drained. The amount of water w (in thousands of gallons) remaining in the pool at time t (in hours) after the draining begins is $w = 600 - 20t$. **See margin.**

Elaborate

8. The inverse function undoes the operations of the original function. It performs the inverse operation of each original operation, and in the reverse order.

7. What must be true about a function for its inverse to be a function? **The function must be one-to-one.**

8. A function rule indicates the operations to perform on an input to produce an output. What is the relationship between these operations and the operations indicated by the inverse function? **See above.**

9. How can you use composition to verify that two functions $f(x)$ and $g(x)$ are inverse functions? **See margin.**

10. Describe a real-world situation modeled by a linear function for which it makes sense to find an inverse function. Give an example of how the inverse function might also be useful. **See margin.**

11. **Essential Question Check-In** What is an inverse relation?

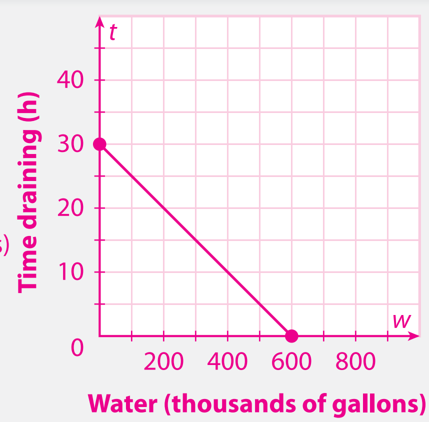
It is a relation that reverses all the pairings of some other relation. If a relation pairs an input x with an output y, then the inverse relation pairs an input y with an output x.

Answers

6. The range of $w = 600 - 20t$ is $\{w \mid 0 \leq w \leq 600\}$. This is the domain of the inverse function.

The inverse function is $-\frac{1}{20}w + 30 = t$.

The inverse function gives the time (in hours) the pool has been draining as a function of the amount of water (in thousands of gallons) remaining in the pool.



Elaborate

Peer-to-Peer Discussion

Ask students to discuss with a partner what must be true about the graph of a function in order for the inverse to be a function. The graph must pass a horizontal line test. That is, no two points of the graph can lie on the same horizontal line.

Questioning Strategies

If you find the inverse of the inverse of a function, will it necessarily be a function? Explain. Yes. The inverse of the inverse of a function is the original function.

Summarize the Lesson

How do you find the inverse of a function, and how is the original function related to its inverse? To find the inverse of a function, replace $f(x)$ with y, solve for x, and then switch x and y. The output of a function is the input of the inverse function. The output of the inverse function is the input of the function. Because the coordinates of the ordered pairs are reversed, the graphs of the function and the inverse function are reflections across the line $y = x$.

Answers

9. Find $f(g(x))$ and $g(f(x))$. If $f(g(x)) = x$ and $g(f(x)) = x$, then $f(x)$ and $g(x)$ are inverse functions.

10. Sample answer: Grapes are \$1.80 per pound at the grocery store. The equation $C = 1.8w$ gives the cost C (in dollars) for grapes with a weight w (in pounds). The inverse function is $w = \frac{C}{1.8}$. It gives the weight of the grapes purchased as a function of their cost. You could use the inverse, for example, to find what weight of grapes you could buy for \$5.

Evaluate

Assignment Guide

Level	Concepts and Skills	Practice
Basic	Explore	Exercises 1–4
	Example 1	Exercises 5–6
	Example 2	Exercise 11
	H.O.T.	Exercise 18
Average	Explore	Exercises 3–4
	Example 1	Exercises 7–8
	Example 2	Exercises 11–14
	H.O.T.	Exercises 17–18
Advanced	Explore	N/A
	Example 1	Exercises 9–10
	Example 2	Exercises 13–15
	H.O.T.	Exercises 16–18

Real World Problems

Avoid Common Errors

Some students may have difficulty reflecting points in the line $y = x$. For these students, you may want to review the concept of reflection, and encourage them to use tracing paper, to fold paper, or to use a reflecting tool to correctly draw the reflection.

Questioning Strategies

When finding the inverse of a function, why do you need to switch x and y after solving the equation for x? because the y-values of the original function become the x-values in the inverse, and vice-versa

Visual Cues

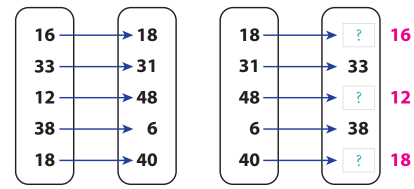
It may be useful to note that when you are graphing an inverse, the y-intercepts of the original function become the x-intercepts of the inverse and vice-versa. So if you are graphing a line, it is easy to quickly establish the two points necessary to determine the inverse.

Evaluate: Homework and Practice

Personal Math Trainer
• Online Homework
• Hints and Help
• Extra Practice

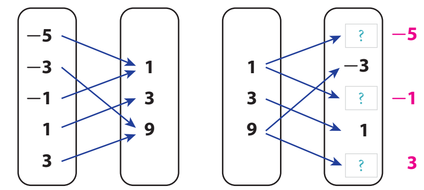
The mapping diagrams show a function and its inverse. Complete the diagram for the inverse of the function. Then tell whether the inverse is a function, and explain your reasoning.

1.



The inverse is a function, because for each input there is only one output.

2.



The inverse is not a function, because the inputs 1 and 9 each have two different outputs.

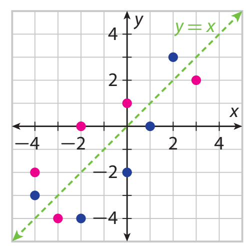
Write the inverse of the given function as a set of ordered pairs and then graph the function and its inverse on a coordinate plane.

3. Function:

{(−4, −3), (−2, −4), (0, −2), (1, 0), (2, 3)}

Inverse of function:

{(−3, −4), (−4, −2), (−2, 0), (0, 1), (3, 2)}

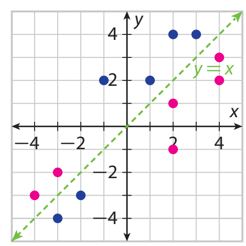


4. Function:

{(−3, −4), (−2, −3), (−1, 2), (1, 2), (2, 4), (3, 4)}

Inverse of function:

{(−4, −3), (−3, −2), (2, −1), (2, 1), (4, 2), (4, 3)}



Find the inverse function $f^{-1}(x)$ for the given function $f(x)$.

5. $f(x) = 4x - 8$ $f^{-1}(x) = \frac{x+8}{4}$

6. $f(x) = \frac{x}{3}$ $f^{-1}(x) = 3x$

7. $f(x) = \frac{x+1}{6}$ $f^{-1}(x) = 6x - 1$

8. $f(x) = -0.75x$ $f^{-1}(x) = -\frac{4}{3}x$

Exercise	Depth of Knowledge (D.O.K.)	Math Processes and Practices
1–4	**1** Recall of Information	**MPP2** Abstract and Quantitative Reasoning
5–8	**2** Skills/Concepts	**MPP5** Using Mathematical Tools
9–10	**2** Skills/Concepts	**MPP4** Mathematical Modeling
11–12	**2** Skills/Concepts	**MPP1** Problem Solving
13–15	**2** Skills/Concepts	**MPP2** Abstract and Quantitative Reasoning
16–17	**3** Strategic Thinking H.O.T.	**MPP6** Using Precise Mathematical Language
18	**3** Strategic Thinking H.O.T.	**MPP4** Mathematical Modeling

Find the inverse function $f^{-1}(x)$ for the given function $f(x)$. Use composition to verify that the functions are inverses. Then graph the function and its inverse. **9–10. See margin.**

9. $f(x) = -3x + 3$

10. $f(x) = \frac{2}{5}x - 2$

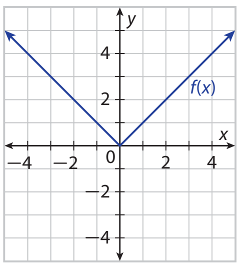
For the given function, determine the domain of the inverse function. Then find an equation for the inverse function, and graph it. Interpret the meaning of the inverse function.

11. **Geometry** The equation $A = \frac{1}{2}(20)h$ gives the area A (in square inches) of a triangle with a base of 20 inches as a function of its height h (in inches). **See below.**

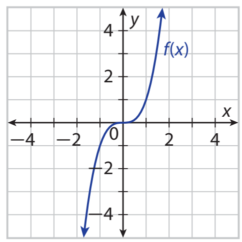
12. The label on a gallon of paint says that it will cover from 250 square feet to 450 square feet depending on the surface that is being painted. A painter has 12 gallons of paint on hand. The equation $A = 12c$ gives the area A (in square feet) that the 12 gallons of paint will cover if applied at a coverage rate c (in square feet per gallon). **See margin.**

The graph of a function is given. Tell whether the function's inverse is a function, and explain your reasoning. If the inverse is not a function, tell how can you restrict the domain of the function so that its inverse is a function.

13.



14.



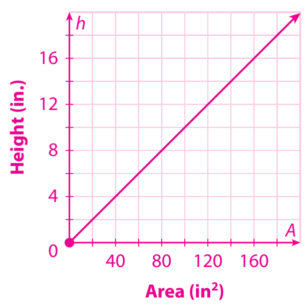
The function's inverse is a function, because the function is one-to-one.

The function's inverse is not a function, because the function is many-to-one. If the domain is restricted to $\{x|x \le 0\}$, $\{x|x \ge 0\}$, or some subset of these sets, the function becomes one-to-one, and the inverse is a function.

11. The domain of the inverse function is $\{A|A \ge 0\}$.

The inverse function is $h = \frac{A}{10}$.

The inverse function gives the height (in inches) of a triangle with a base of 20 inches as a function of the area (in square inches) of the triangle.

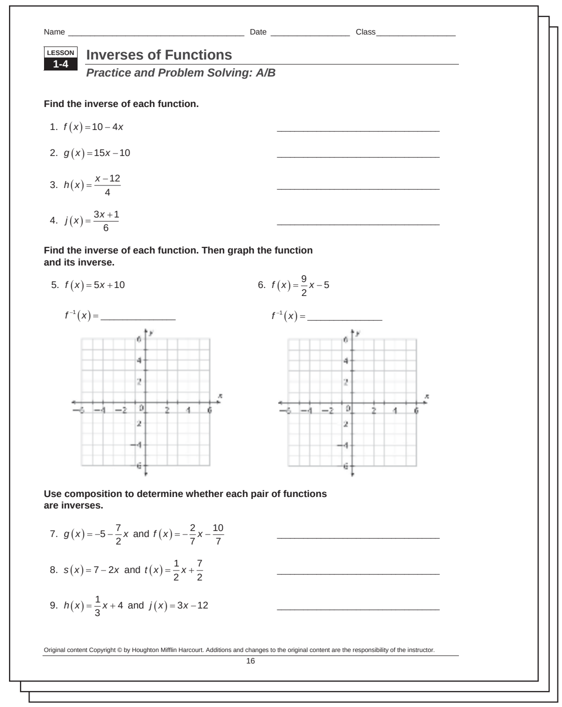


Journal

Have students describe a real-world situation in which it would be useful to find an inverse in order to answer a question.

Online Resources

- Practice and Problem Solving (three forms)
- Reteach
- Reading Strategies
- Success for English Learners



Name ______ Date ______ Class ______

LESSON 1-4 Inverses of Functions
Practice and Problem Solving: A/B

Find the inverse of each function.

1. $f(x) = 10 - 4x$
2. $g(x) = 15x - 10$
3. $h(x) = \frac{x-12}{4}$
4. $j(x) = \frac{3x+1}{6}$

Find the inverse of each function. Then graph the function and its inverse.

5. $f(x) = 5x + 10$ $f^{-1}(x) =$ ______
6. $f(x) = \frac{9}{2}x - 5$ $f^{-1}(x) =$ ______

Use composition to determine whether each pair of functions are inverses.

7. $g(x) = -5 - \frac{7}{2}x$ and $f(x) = -\frac{2}{7}x - \frac{10}{7}$
8. $s(x) = 7 - 2x$ and $t(x) = \frac{1}{2}x + \frac{7}{2}$
9. $h(x) = \frac{1}{3}x + 4$ and $j(x) = 3x - 12$

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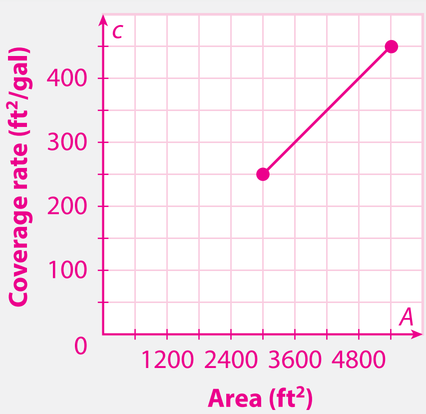
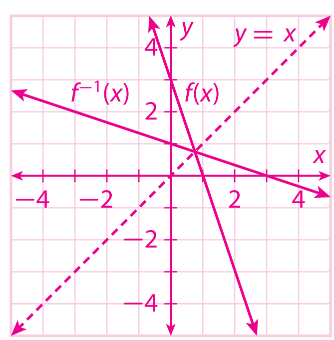
Answers

12. The domain of the inverse function is $\{A|3000 \le A \le 5400\}$.
The inverse function is $c = \frac{A}{12}$.
The inverse function gives the coverage rate (in square feet per gallon) at which 12 gallons of paint must be applied as a function of the area covered (in square feet).

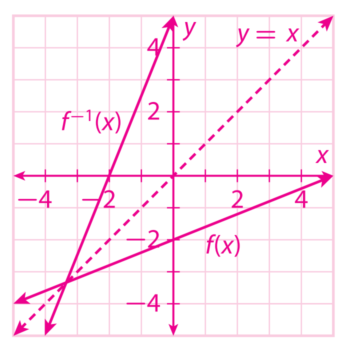


Answers

9. $f^{-1}(x) = -\frac{1}{3}x + 1$



10. $f^{-1}(x) = \frac{5}{2}x + 5$



Connect Vocabulary EL

Some students may not be familiar with the term *femur* mentioned in this problem. Show students a diagram of a human skeleton. Explain that the femur is the only bone in the thigh and it is the longest, heaviest, and strongest bone in the human body.

Integrate Math Processes and Practices

Focus on Using and Evaluating Logical Reasoning

MPP3 Discuss with students whether or not this data set is suitable for representation by a linear regression model. Ask them to tell how they might go about determining the best type of regression model to use for this particular set of data.

Avoid Common Errors

Students may not divide the entire expression $f + 43.6$ by 0.533. Have students tell the order of the operations in the function (multiply, then subtract) and then tell the reverse of these operations (add, then divide). Since $f + 43.6$ cannot be simplified before dividing, have students put parentheses around the expression before dividing by 0.533: $h_m = \frac{(f + 43.6)}{0.533}$.

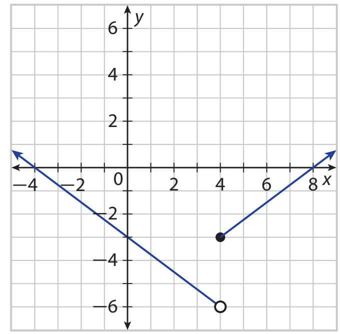
Answers

16. The line $y = x$ is its own inverse. Also, any line that is perpendicular to the line $y = x$ is its own inverse, since it will be its own reflection across that line. So, any linear function of the form $y = -x + b$ where b is a real number is its own inverse.

Lesson Performance Task Scoring Rubric

Points	Criteria
2	Student correctly solves the problem and explains his/her reasoning.
1	Student shows good understanding of the problem but does not fully solve or explain his/her reasoning.
0	Student does not demonstrate understanding of the problem.

15. **Multiple Response** Identify the domain intervals over which the inverse of the graphed function is also a function. Select all that apply.



A. $[4, +\infty)$ D. $(-\infty, +\infty)$ G. $(4, 8)$

B. $(0, +\infty)$ E. $(-\infty, 4]$ H. $(8, +\infty)$

C. $[-4, +\infty)$ F. $(-\infty, 4)$ I. $(0, 8]$

A, B, F, G and H.

H.O.T. Focus on Higher Order Thinking

16. **Draw Conclusions** Identify all linear functions that are their own inverse. **See margin.**

17. **Make a Conjecture** Among linear functions (excluding constant functions), quadratic functions, absolute value functions, and exponential functions, which types of function do you have to restrict the domain for the inverse to be a function? Explain. **See below.**

18. **Find the Error** A student was asked to find the inverse of $f(x) = 2x + 1$. The student's work is shown. Explain why the student is incorrect and what the student should have done to get the correct answer.

> The function $f(x) = 2x + 1$ involves two operations: multiplying by 2 and adding 1. The inverse operations are dividing by 2 and subtracting 1. So, the inverse function is $f^{-1}(x) = \frac{x}{2} - 1$.

See Additional Answers.

Lesson Performance Task

In an anatomy class, a student measures the femur of an adult male and finds the length of the femur to be 50.0 cm. The student is then asked to estimate the height of the male that the femur came from.

The table shows the femur lengths and heights of some adult males and females. Using a graphing calculator, perform linear regression on the data to obtain femur length as a function of height (one function for adult males, one for adult females). Then find the inverse of each function. Use the appropriate inverse function to find the height of the adult male and explain how the inverse functions would be helpful to a forensic scientist. **See Additional Answers.**

Femur Length (cm)	30	38	46	54	62
Male Height (cm)	138	153	168	183	198
Female Height (cm)	132	147	163	179	194

17. You have to restrict the domains of quadratic functions and absolute value functions, because these functions are many-to-one functions. For instance, the quadratic function $f(x) = x^2$ pairs both −2 and 2 with 4, and the absolute value function $f(x) = |x|$ pairs both −2 and 2 with 2. On the other hand, linear functions (excluding constant functions) and exponential functions are one-to-one functions, so their domains do not need to be restricted.

Extension Activity

Have students measure the femur lengths (cm) and heights (cm) of 5 male students and record the data in a table. Then have students answer the questions in the Performance Task using their data instead of the data given. Have students explain how their results are similar or different from those found in the Performance Task, and also to note the reasons why the measurements cannot be as accurate. Discuss how technology such as an x-ray, CT scan, or MRI, could be used to improve accuracy.