1.3 Transformations of Function Graphs

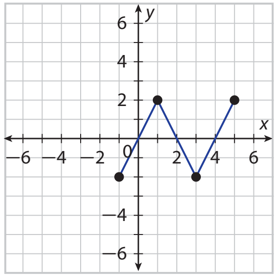
Essential Question: What are the ways you can transform the graph of the function f(x)?

Resource Locker

Explore 1 Investigating Translations of Function Graphs

You can transform the graph of a function in various ways. You can translate the graph horizontally or vertically, you can stretch or compress the graph horizontally or vertically, and you can reflect the graph across the x-axis or the y-axis. How the graph of a given function is transformed is determined by the way certain numbers, called **parameters**, are introduced in the function.

The graph of $f(x)$ is shown. Copy this graph and use the same grid for the exploration.



Ⓐ First graph $g(x) = f(x) + k$ where k is the parameter. Let $k = 4$ so that $g(x) = f(x) + 4$. Complete the input-output table and then graph $g(x)$. In general, how is the graph of $g(x) = f(x) + k$ related to the graph of $f(x)$ when k is a positive number?

x	$f(x)$	$f(x) + 4$
−1	−2	2
1	2	6
3	−2	? **2**
5	2	? **6**

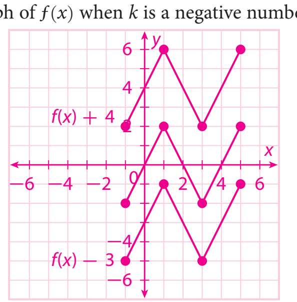
For $k > 0$, the graph of $g(x) = f(x) + k$ is the graph of $f(x)$ translated up k units.

See graph below.

Ⓑ Now try a negative value of k in $g(x) = f(x) + k$. Let $k = -3$ so that $g(x) = f(x) - 3$. Complete the input-output table and then graph $g(x)$ on the same grid. In general, how is the graph of $g(x) = f(x) + k$ related to the graph of $f(x)$ when k is a negative number?

x	$f(x)$	$f(x) - 3$
−1	−2	−5
1	2	−1
3	−2	? **−5**
5	2	? **−1**

A–B.



For $k < 0$, the graph of $g(x) = f(x) + k$ is the graph of $f(x)$ translated down $|k|$ units.

Professional Development

Math Background

Transformations change the graph of a function. When students understand the basic transformations (translation, reflection, stretch, and compression), they are better able to understand how to write the equation of a graph, and how to identify the graph of a function that has been transformed.

Transformations of Function Graphs

Learning Objective

Students will investigate translations, stretches, compressions, and reflections of function graphs, and transform the graph of the parent quadratic function.

Math Processes and Practices

MPP2 Abstract and Quantitative Reasoning

Language Objective

Identify graphs of odd and even functions and justify reasoning with a partner.

Online Resources

An extra example for each Explain section is available online.

Engage

Essential Question: What are the ways you can transform the graph of the function $y = f(x)$?

Possible answer: The parameter h in $y = f(x - h)$ produces a horizontal translation of the graph of $y = f(x)$. The parameter k in $y = f(x) + k$ produces a vertical translation of the graph of $y = f(x)$. The parameter a in $y = af(x)$ produces a vertical stretch/compression of the graph of $y = f(x)$ and may also produce a reflection across the x-axis. The parameter b in $y = f\left(\frac{1}{b}x\right)$ produces a horizontal stretch/compression of the graph of $y = f(x)$ and may also produce a reflection across the y-axis.

Preview: Lesson Performance Task

View the online Engage. Discuss the photo and the guidelines someone might need to follow when designing a chair. Then preview the Lesson Performance Task.

Explore 1

Investigating Translations of Function Graphs

Integrate Technology

Students have the option of completing the activity either in the book or online.

Avoid Common Errors

Students may confuse directions in horizontal translations. Emphasize that h is the number *subtracted* from x. For example, in $f(x-3)$, the value of h is 3, a positive number, and therefore the translation is to the right.

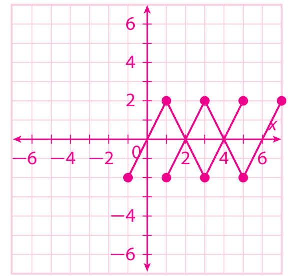
Questioning Strategies

Given the graph of a function $f(x)$, and the graph of the image of the function after a translation, how can you determine the rule for the function represented by the image? You can select a particular point on the original graph (such as an endpoint of a segment, or a local maximum or minimum point), and see how its image was obtained. If the image is above or below the original point, the rule will involve adding or subtracting the number of units it was translated to $f(x)$. If the image is to the left or right of the original point, the rule will involve adding or subtracting the number of units it was translated to x in $f(x)$.

C Now graph $g(x) = f(x-h)$ where h is the parameter. Let $h = 2$ so that $g(x) = f(x-2)$. Complete the mapping diagram and then graph $g(x)$ with $f(x)$. (To complete the mapping diagram, you need to find the inputs for g that produce the inputs for f after you subtract 2. Work backward from the inputs for f to the missing inputs for g by adding 2.) In general, how is the graph of $g(x) = f(x-h)$ related to the graph of $f(x)$ when h is a positive number?

Input for g	Input for f	Output for f	Output for g
1	−1	−2	−2
3	1	2	2
5 ?	3	−2	−2
7 ?	5	2	2

-2



For $h > 0$, the graph of $g(x) = f(x-h)$ is the graph of $f(x)$ translated right h units.

D **Make a Conjecture** How would you expect the graph of $g(x) = f(x-h)$ to be related to the graph of $f(x)$ when h is a negative number?

For $h < 0$, the graph of $g(x) = f(x-h)$ is the graph of $f(x)$ translated left $|h|$ units.

Reflect

1. The domain remains the same, and the range changes from $[y_1, y_2]$ to $[y_1 + k, y_2 + k]$.

1. Suppose a function $f(x)$ has a domain of $[x_1, x_2]$ and a range of $[y_1, y_2]$. When the graph of $f(x)$ is translated vertically k units where k is either positive or negative, how do the domain and range change?

2. Suppose a function $f(x)$ has a domain of $[x_1, x_2]$ and a range of $[y_1, y_2]$. When the graph of $f(x)$ is translated horizontally h units where h is either positive or negative, how do the domain and range change? **See below.**

3. You can transform the graph of $f(x)$ to obtain the graph of $g(x) = f(x-h) + k$ by combining transformations. Predict what will happen by completing the table.

Sign of h	Sign of k	Transformations of the Graph of $f(x)$
+	+	Translate right h units and up k units.
+	−	?
−	+	?
−	−	?

$h+$, $k-$: Translate right h units and down $|k|$ units.

$h-$, $k+$: Translate left $|h|$ units and up k units.

$h-$, $k-$: Translate left $|h|$ units and down $|k|$ units.

2. The domain changes from $[x_1, x_2]$ to $[x_1 - h, x_2 - h]$, and the range remains the same.

Collaborative Learning

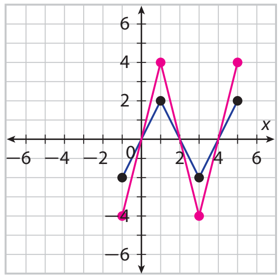
Peer-to-Peer Activity

Have students work in pairs. Provide students with three or four functions, and have them use a graphing calculator to explore how changes to the various parameters in each function affect the graph of the function. Once they start to make the connections, encourage them to try and predict each change before graphing the transformation.

Explore 2 Investigating Stretches and Compressions of Function Graphs

In this activity, you will consider what happens when you multiply by a positive parameter inside or outside a function. Throughout, you will use the same function $f(x)$ that you used in the previous activity.

A First graph $g(x) = a \cdot f(x)$ where a is the parameter. Let $a = 2$ so that $g(x) = 2f(x)$. Complete the input-output table and then graph $g(x)$ with $f(x)$. In general, how is the graph of $g(x) = a \cdot f(x)$ related to the graph of $f(x)$ when a is greater than 1?



x	$f(x)$	$2f(x)$
−1	−2	−4
1	2	4
3	−2	? −4
5	2	? 4

For $a > 1$, the graph of $g(x) = a \cdot f(x)$ is the graph of $f(x)$ stretched vertically (away from the x-axis) by a factor of a.

B Now try a value of a between 0 and 1 in $g(x) = a \cdot f(x)$. Let $a = \frac{1}{2}$ so that $g(x) = \frac{1}{2}f(x)$. Complete the input-output table and then graph $g(x)$ with $f(x)$. In general, how is the graph of $g(x) = a \cdot f(x)$ related to the graph of $f(x)$ when a is a number between 0 and 1?

x	$f(x)$	$\frac{1}{2}f(x)$
−1	−2	−1
1	2	1
3	−2	? −1
5	2	? 1

See graph in margin.

For $0 < a < 1$, the graph of $g(x) = a \cdot f(x)$ is the graph of $f(x)$ compressed vertically (toward the x-axis) by a factor of a.

C Now graph $g(x) = f\left(\frac{1}{b} \cdot x\right)$ where b is the parameter. Let $b = 2$ so that $g(x) = f\left(\frac{1}{2}x\right)$. Complete the mapping diagram and then graph $g(x)$ with $f(x)$. (To complete the mapping diagram, you need to find the inputs for g that produce the inputs for f after you multiply by $\frac{1}{2}$. Work backward from the inputs for f to the missing inputs for g by multiplying by 2.) In general, how is the graph of $g(x) = f\left(\frac{1}{b}x\right)$ related to the graph of $f(x)$ when b is a number greater than 1?

Input for g ($\cdot \frac{1}{2}$)	Input for f	Output for f	Output for g
−2	−1	−2	−2
2	1	2	2
? 6	3	−2	−2
? 10	5	2	2

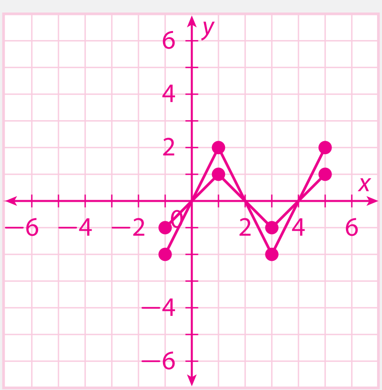
See graph in margin.

For $b > 1$, the graph of $g(x) = f\left(\frac{1}{b} \cdot x\right)$ is the graph of $f(x)$ stretched horizontally (away from the y-axis) by a factor of b.

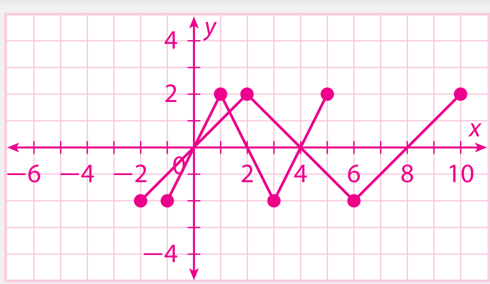
D **Make a Conjecture** How would you expect the graph of $g(x) = f\left(\frac{1}{b} \cdot x\right)$ to be related to the graph of $f(x)$ when b is a number between 0 and 1? **See margin.**

Answers

B.



C.



D. For $0 < b < 1$, the graph of $g(x) = f\left(\frac{1}{b} \cdot x\right)$ is the graph of $f(x)$ compressed horizontally (toward the y-axis) by a factor of b.

Explore 2

Investigating Stretches and Compressions of Function Graphs

Questioning Strategies

On the coordinate plane, what is the difference between a vertical stretch and a vertical compression? In a vertical stretch, the points of the graph are pulled *away* from the x-axis. In a vertical compression, they are pulled *toward* the x-axis.

On the coordinate plane, what is the difference between a horizontal stretch and a vertical stretch? In a horizontal stretch, the points of the graph are pulled away from the y-axis. In a vertical stretch, the points of the graph are pulled away from the x-axis.

Integrate Technology

A graphing calculator can be used to explore the effects of different values of b in the function $g(x) = f\left(\frac{1}{b}x\right)$ on the graph of $f(x)$. Choose a simple function for $f(x)$, graph the function, and have students suggest different values for b. Graph the transformed functions, and have students compare the graphs to see how changing the parameter affects the graph.

Integrate Math Processes and Practices Focus on Abstract and Quantitative Reasoning

MPP2 Prompt students to recognize that when the graph of a function passes through the origin, a transformation involving a stretch or a compression of the function will not affect the point at the origin. Ask students to justify how this is possible, when all points on either side of the origin *are* affected.

Explore 3

Investigating Reflections of Function Graphs

Integrate Math Processes and Practices

Focus on Using Mathematical Tools

MPP5 To help students understand the symmetry of the graphs of even and odd functions, have students use a graphing calculator to graph $f(x) = x^2$ and $g(x) = f(-x) = (-x)^2$ to see that the two graphs coincide. This is a consequence of the fact that replacing x with $-x$ in a function rule causes the graph to be reflected across the y-axis. Since the graph of $f(x) = x^2$ is symmetric with respect to the y-axis, it is unaffected when x is replaced with $-x$.

Also have students graph $f(x) = x^3$, $g(x) = f(-x) = (-x)^3$, and $h(x) = -f(x) = -x^3$, and observe that the graphs of $g(x)$ and $h(x)$ coincide. The graph of $g(x)$ is a reflection of the graph of $f(x)$ across the y-axis, while the graph of $h(x)$ is a reflection of the graph of $f(x)$ across the x-axis. The graphs of $g(x)$ and $h(x)$ coincide because reflecting the graph of $f(x)$ across both axes does not change the graph, which is another way of saying that the graph of $f(x)$ has 180° rotational symmetry about the origin.

Questioning Strategies

How do the domain and range of the function $g(x) = f(-x)$ compare to the domain and range of $f(x)$? The domain of $g(x)$ consists of the opposites of the elements in the domain of $f(x)$. The range values are the same.

Reflect **4. The domain remains the same, and the range changes from $[y_1, y_2]$ to $[ay_1, ay_2]$.**

4. Suppose a function $f(x)$ has a domain of $[x_1, x_2]$ and a range of $[y_1, y_2]$. When the graph of $f(x)$ is stretched or compressed vertically by a factor of a, how do the domain and range change?

5. You can transform the graph of $f(x)$ to obtain the graph of $g(x) = a \cdot f(x-h) + k$ by combining transformations. Predict what will happen by completing the table.

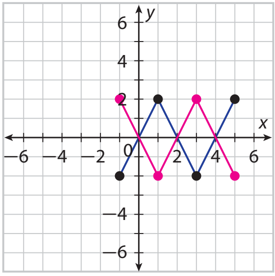
Value of a	Transformations of the Graph of $f(x)$
$a > 1$	Stretch vertically by a factor of a, and translate h units horizontally and k units vertically.
$0 < a < 1$	**See margin.** ?

6. You can transform the graph of $f(x)$ to obtain the graph of $g(x) = f\left(\frac{1}{b}(x-h)\right) + k$ by combining transformations. Predict what will happen by completing the table.

Value of b	Transformations of the Graph of $f(x)$
$b > 1$	Stretch horizontally by a factor of b, and translate h units horizontally and k units vertically.
$0 < b < 1$	**See margin.** ?

Explore 3 Investigating Reflections of Function Graphs

When the parameter in a stretch or compression is negative, another transformation called a *reflection* is introduced. Examining reflections will also tell you whether a function is an *even function* or an *odd function*. An **even function** is one for which $f(-x) = f(x)$ for all x in the domain of the function, while an **odd function** is one for which $f(-x) = -f(x)$ for all x in the domain of the function. A function is not necessarily even or odd; it can be neither.



(A) First graph $g(x) = a \cdot f(x)$ where $a = -1$. Complete the input-output table and then graph $g(x) = -f(x)$ with $f(x)$. In general, how is the graph of $g(x) = -f(x)$ related to the graph of $f(x)$?

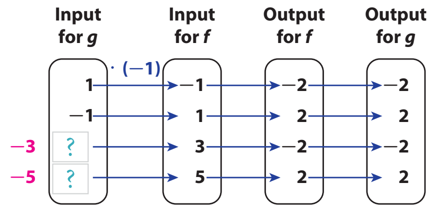
x	$f(x)$	$-f(x)$
−1	−2	2
1	2	−2
3	−2	? **2**
5	2	? **−2**

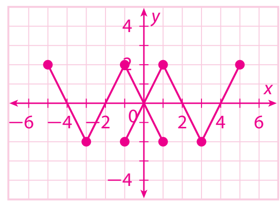
The graph of $g(x) = -f(x)$ is a reflection of the graph of $f(x)$ across the x-axis.

Answers

5. Compress vertically by a factor of a, and translate h units horizontally and k units vertically.
6. Compress horizontally by a factor of b, and translate h units horizontally and k units vertically.

(B) Now graph $g(x) = f\left(\frac{1}{b} \cdot x\right)$ where $b = -1$. Complete the input-output table and then graph $g(x) = f(-x)$ with $f(x)$. In general, how is the graph of $g(x) = f(-x)$ related to the graph of $f(x)$?





The graph of $g(x) = -f(x)$ is a reflection of the graph of $f(x)$ across the x-axis.

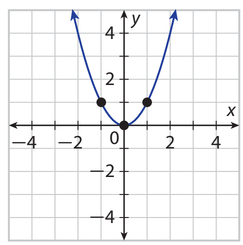
Reflect 7–9. See margin.

7. **Discussion** Suppose a function $f(x)$ has a domain of $[x_1, x_2]$ and a range of $[y_1, y_2]$. When the graph of $f(x)$ is reflected across the x-axis, how do the domain and range change?

8. For a function $f(x)$, suppose the graph of $f(-x)$, the reflection of the graph of $f(x)$ across the y-axis, is identical to the graph of $f(x)$. What does this tell you about $f(x)$? Explain.

9. Is the function whose graph you reflected across the axes in Steps A and B an even function, an odd function, or neither? Explain.

Explain 1 Transforming the Graph of the Parent Quadratic Function



You can use transformations of the graph of a basic function, called a *parent function*, to obtain the graph of a related function. To do so, focus on how the transformations affect reference points on the graph of the parent function.

For instance, the parent quadratic function is $f(x) = x^2$. The graph of this function is a U-shaped curve called a *parabola* with a turning point, called a *vertex*, at (0, 0). The vertex is a useful reference point, as are the points (−1, 1) and (1, 1).

Example 1 **Describe how to transform the graph of $f(x) = x^2$ to obtain the graph of the related function $g(x)$. Then draw the graph of $g(x)$.**

(A) $g(x) = -3f(x - 2) - 4$

Parameter and Its Value	Effect on the Parent Graph
$a = -3$	vertical stretch of the graph of $f(x)$ by a factor of 3 and a reflection across the x-axis
$b = 1$	Since $b = 1$, there is no horizontal stretch or compression.
$h = 2$	horizontal translation of the graph of $f(x)$ to the right 2 units
$k = -4$	vertical translation of the graph of $f(x)$ down 4 units

Differentiate Instruction

Visual Cues

Suggest that students write the general function $g(x) = af(x - h)^2 + k$, (or $f(x) = a(x - h)^2 + k$, depending on the context) above the specific function they are analyzing in order to correctly identify the parameters in the transformation.

Communicating Math

When analyzing transformed functions, have students list each parameter and its value, and then write a short phrase, such as "shift 5 units to the left" or "reflect across the x-axis" to indicate the meaning of each parameter. This may make it easier for the student to then draw the graph of the function.

Explain 1

Transforming the Graph of the Parent Quadratic Function

Questioning Strategies

When drawing the graph of a transformation of the function $f(x) = x^2$ that involves a reflection across the x-axis, a horizontal translation, and a vertical stretch, in which order should the transformations be applied to the graph of the parent function? Explain. The order does not matter. The graph of the new function will be the same no matter what the order.

When drawing the graph of a transformation of the function $f(x) = x^2$ that involves a reflection, a vertical translation, and a horizontal compression, in which order should the transformations be applied to the graph of the parent function? Explain. The reflection and compression need to be applied before the vertical translation. The new function is a vertical translation of the graph of $f(x) = ax^2$.

Answers

7. The domain remains the same, and the range changes from $[y_1, y_2]$ to $[-y_2, -y_1]$.
8. Because $f(x) = f(-x)$, $f(x)$ must be an even function.
9. It is neither an even function nor an odd function, because the graph of $f(-x)$ in Step B is not identical to the graph of $f(x)$ nor is it identical to the graph of $-f(x)$ in Step A.

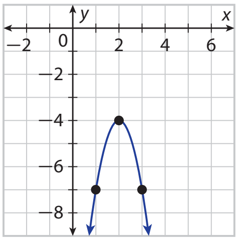
Integrate Math Processes and Practices
Focus on Abstract and Quantitative Reasoning

MPP1 Discuss with students how the attributes of a function (such as domain, range, maximum or minimum values, and intervals over which the function is increasing or decreasing) can be determined from the function written in the form $g(x) = af(x - h)^2 + k$, where $f(x)$ is the function $f(x) = x^2$.

Applying these transformations to a point (x, y) on the parent graph results in the point $(x + 2, -3y - 4)$. The table shows what happens to the three reference points on the graph of $f(x)$.

Point on the Graph of $f(x)$	Corresponding Point on $g(x)$
$(-1, 1)$	$(-1 + 2, -3(1) - 4) = (1, -7)$
$(0, 0)$	$(0 + 2, -3(0) - 4) = (2, -4)$
$(1, 1)$	$(1 + 2, -3(1) - 4) = (3, -7)$

Use the transformed reference points to graph $g(x)$.



B $g(x) = f\left(\frac{1}{2}(x + 5)\right) + 2$

Parameter and Its Value	Effect on the Parent Graph
$a = \boxed{1}$	Since $a = 1$, there is no vertical stretch, no vertical compression, and no reflection across the x-axis.
$b = \boxed{2}$	The parent graph is stretched horizontally by a factor of 2. There is no reflection across the y-axis.
$h = \boxed{-5}$	The parent graph is translated −5 units horizontally.
$k = \boxed{2}$	The parent graph is translated 2 units vertically.

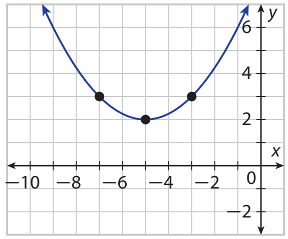
Applying these transformations to a point on the parent graph results in the point $(2x - 5, y + 2)$. The table shows what happens to the three reference points on the graph of $f(x)$.

Point on the Graph of $f(x)$	Corresponding Point on the Graph of $g(x)$
$(-1, 1)$	$(2(-1) - 5, 1 + 2) = \left(\boxed{-7}, \boxed{3}\right)$
$(0, 0)$	$(2(0) - 5, 0 + 2) = \left(\boxed{-5}, \boxed{2}\right)$
$(1, 1)$	$(2(1) - 5, 1 + 2) = \left(\boxed{-3}, \boxed{3}\right)$

Language Support EL
Connect Vocabulary

Have students work individually and then with a partner on this activity. Give each student pictures on paper or graphing calculators showing images of the graphs of even and odd functions. Have each student identify whether the graph is an even or odd function. Once they decide, they should work with a partner. Each partner has to agree or disagree with the choices made by the other person, and explain why he/she agrees or disagrees. In their explanations, encourage the use of the terms *reflection, x-axis, y-axis, coincides with graph of f.*

Use the transformed reference points to graph $g(x)$.



Reflect

10. Is the function $f(x) = x^2$ an even function, an odd function, or neither? Explain. **Even; $f(-x) = f(x)$ for all values of x in its domain.**

11. The graph of the parent quadratic function $f(x) = x^2$ has the vertical line $x = 0$ as its axis of symmetry. Identify the axis of symmetry for each of the graphs of $g(x)$ in Parts A and B. Which transformation(s) affect the location of the axis of symmetry? **In Part A, the axis of symmetry is $x = 2$. In Part B, the axis of symmetry is $x = -5$. Only a horizontal translation affects the location of the axis of symmetry.**

Your Turn

12. Describe how to transform the graph of $f(x) = x^2$ to obtain the graph of the related function $g(x) = f\big(-4(x - 3)\big) + 1$. Then draw the graph of $g(x)$. **See margin.**

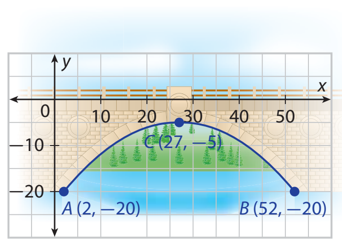
Explain 2 Modeling with a Quadratic Function

You can model real-world objects that have a parabolic shape using a quadratic function. In order to fit the function's graph to the shape of the object, you will need to determine the values of the parameters in the function $g(x) = a \cdot f\left(\frac{1}{b}(x - h)\right) + k$ where $f(x) = x^2$. Note that because $f(x)$ is simply a squaring function, it's possible to pull the parameter b outside the function and combine it with the parameter a. Doing so allows you to model real-objects using $g(x) = a \cdot f(x - h) + k$, which has only three parameters.

When modeling real-world objects, remember to restrict the domain of $g(x) = a \cdot f(x - h) + k$ to values of x that are based on the object's dimensions.

Example 2

An old stone bridge over a river uses a parabolic arch for support. In the illustration shown, the unit of measurement for both axes is feet, and the vertex of the arch is point C. Find a quadratic function that models the arch, and state the function's domain.



Analyze Information

Identify the important information.

- The shape of the arch is a parabola.
- The vertex of the parabola is $C(27, -5)$.
- Two other points on the parabola are $A(2, -20)$ and $B(52, -20)$.

Formulate a Plan

You want to find the values of the parameters a, h, and k in $g(x) = a \cdot f(x - h) + k$ where $f(x) = x^2$. You can use the coordinates of point C to find the values of h and k. Then you can use the coordinates of one of the other points to find the value of a.

Explain 2

Modeling with a Quadratic Function

Avoid Common Errors

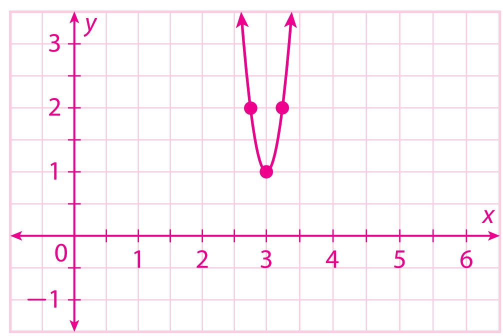
Students may use the wrong signs to indicate the horizontal and vertical translations. Reinforce that the number indicating the horizontal translation must be *subtracted* from x, whereas the number indicating the vertical translation must be *added* to the function.

Questioning Strategies

When modeling a real-world object using a quadratic function, why do you need to restrict the domain of the function? The graph of the function is part of a parabola. It has a left-most point and a right-most point. Thus, the domain consists only of values between, and including, those two points.

Answers

12. The graph of $g(x) = f\big(-4(x - 3)\big) + 1$ is a reflection of the graph of $f(x)$ across the y-axis, a horizontal compression by a factor of $\frac{1}{4}$, and a translation of 3 units to the right and 1 unit up.



Integrate Math Processes and Practices
Focus on Mathematical Modeling

MPP4 Have students brainstorm real-world objects that can be modeled using quadratic functions. Make a list of the objects mentioned, and have students discuss (in general terms) the value of the parameter a in $f(x) = a(x - h)^2 + k$, for each object.

Solve

The vertex of the graph of $g(x)$ is point C, and the vertex of the graph of $f(x)$ is the origin. Point C is the result of translating the origin 27 units to the right and 5 units down. This means that $h = 27$ and $k = -5$. Substituting these values into $g(x)$ gives $g(x) = a \cdot f(x - 27) - 5$. Now substitute the coordinates of point B into $g(x)$ and solve for a.

$g(x) = a \cdot f(x - 27) - 5$ Write the general function.

$g\left(\boxed{52}\right) = a \cdot f(52 - 27) - 5$ Substitute 52 for x.

$-20 = a \cdot f(52 - 27) - 5$ Replace $g(52)$ with -20, the y-value of B.

$-20 = a \cdot f\left(\boxed{25}\right) - 5$ Simplify.

$-20 = a(625) - 5$ Evaluate $f(25)$ for $f(x) = x^2$.

$a = \boxed{-\frac{3}{125}}$ Solve for a.

Substitute the value of a into $g(x)$.

$g(x) = -\frac{3}{125} f(x - 27) - 5$

The arch exists only between points A and B, so the domain of $g(x)$ is $\{x \mid 2 \le x \le 52\}$.

Justify and Evaluate

To justify the answer, verify that $g(2) = -20$.

$g(x) = -\frac{3}{125} f(x - 27) - 5$ Write the function.

$g\left(\boxed{2}\right) = -\frac{3}{125} f\left(\boxed{2} - 27\right) - 5$ Substitute 2 for x.

$= -\frac{3}{125} f\left(\boxed{-25}\right) - 5$ Subtract.

$= -\frac{3}{125} \cdot \left(\boxed{625}\right) - 5$ Evaluate $f(-25)$.

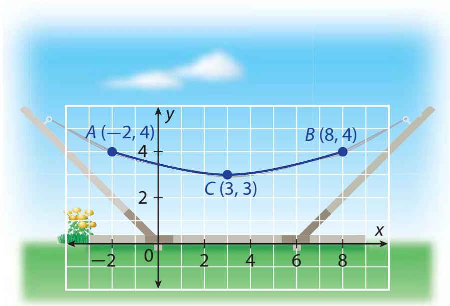
$= -20$ ✓ Simplify.

Your Turn

13. The netting of an empty hammock hangs between its supports along a curve that can be modeled by a parabola. In the illustration shown, the unit of measurement for both axes is feet, and the vertex of the curve is point C. Find a quadratic function that models the hammock's netting, and state the function's domain.

$g(x) = \frac{1}{25} f(x - 3) + 3$

Domain: $\{x \mid -2 \le x \le 8\}$



Elaborate

14–16. See margin.

14. What is the general procedure to follow when graphing a function of the form $g(x) = a \cdot f(x - h) + k$ given the graph of $f(x)$?

15. What are the general steps to follow when determining the values of the parameters a, h, and k in $f(x) = a(x - h)^2 + k$ when modeling a parabolic real-world object?

16. **Essential Question Check-In** How can the graph of a function $f(x)$ be transformed?

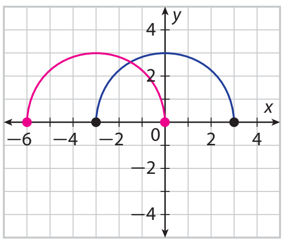
Evaluate: Homework and Practice

Personal Math Trainer

- Online Homework
- Hints and Help
- Extra Practice

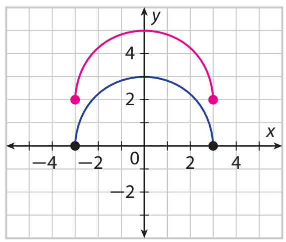
Write $g(x)$ in terms of $f(x)$ after performing the given transformation of the graph of $f(x)$.

1. Translate the graph of $f(x)$ to the left 3 units.



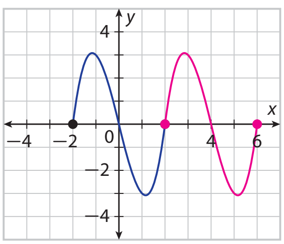
$g(x) = f(x + 3)$

2. Translate the graph of $f(x)$ up 2 units.



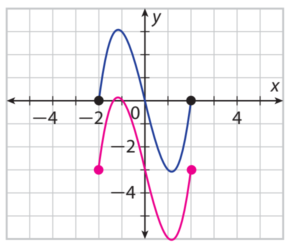
$g(x) = f(x) + 2$

3. Translate the graph of $f(x)$ to the right 4 units.



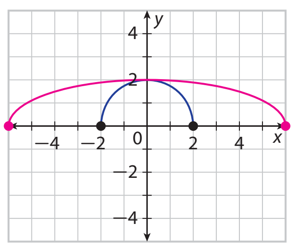
$g(x) = f(x - 4)$

4. Translate the graph of $f(x)$ down 3 units.



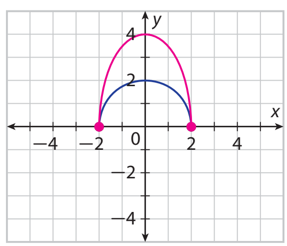
$g(x) = f(x) - 3$

5. Stretch the graph of $f(x)$ horizontally by a factor of 3.



$g(x) = f\left(\frac{1}{3}x\right)$

6. Stretch the graph of $f(x)$ vertically by a factor of 2.



$g(x) = 2f(x)$

Answers

14. Choose reference points on the graph of $f(x)$ and apply the transformations for $g(x)$ to them.
15. To model a parabolic real-world object, first define a coordinate system. Then identify three points on the object; ideally, one of the points should be the vertex. The coordinates of the vertex on the object, (x_v, y_v), are the values of h and k, respectively. Use a second point, (x_1, y_1), on the object to solve $y_1 = a(x_1 - x_v)^2 + y_v$ for a and get $a\frac{y_1 - y_v}{(x_1 - x_v)^2}$. You can use the third point as a check on your results.
16. The graph can be stretched or compressed horizontally or vertically, it can be reflected across the x-axis or y-axis, and it can be translated horizontally or vertically.

Elaborate

Integrate Math Processes and Practices

Focus on Problem Solving

MPP1 Remind students that the real zeros of a function are the x-intercepts of the graph of the function. Have them discuss how the number of real zeros of a function can be determined by knowing the values of a, h, and k in $f(x) = a(x - h)^2 + k$.

Summarize the Lesson

How does each of the following transformations of $f(x) = x^2$ affect the values of a, h, and k in the function $g(x) = af(x - h)^2 + k$?

translation 3 units left $h = -3$

translation 3 units up $k = 3$

vertical stretch by a factor of 3 $a = 3$

reflection across the x-axis $a = -1$

Evaluate

Assignment Guide

Level	Concepts and Skills	Practice
Basic	Explore 1	Exercises 1–2
	Explore 2	Exercises 5–6
	Explore 3	Exercises 9, 10, 13
	Example 1	Exercise 16
	Example 2	Exercise 18
	H.O.T.	Exercise 19
Average	Explore 1	Exercises 3–4
	Explore 2	Exercises 5–8
	Explore 3	Exercises 9–13
	Example 1	Exercises 15–16
	Example 2	Exercise 17
	H.O.T.	Exercise 20
Advanced	Explore 1	N/A
	Explore 2	N/A
	Explore 3	Exercises 11, 12, 14
	Example 1	Exercises 15–16
	Example 2	Exercises 17–18
	H.O.T.	Exercises 19–20

Real World Problems

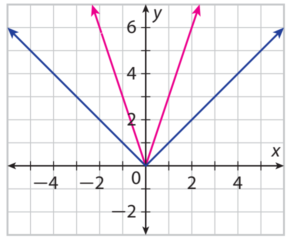
Avoid Common Errors

Students may confuse the concepts of *stretch* and *compression* (both vertical and horizontal). Tell them that they can evaluate the function for a specific value of x, and compare it to the value of $f(x)$. Plotting one or two of these points will provide a visual cue as to the nature of the transformation.

Critical Thinking

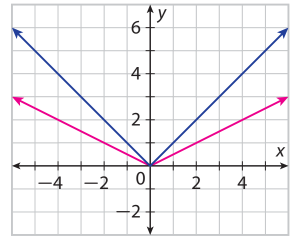
Given a function $f(x)$, how do the functions $g(x) = -f(x)$ and $h(x) = f(-x)$ differ in terms of their effects on the ordered pairs that belong to $f(x)$?
$g(x)$ is the function that negates the $f(x)$ values in each ordered pair; $h(x)$ negates the x-value in each ordered pair.

7. Compress the graph of $f(x)$ horizontally by a factor of $\frac{1}{3}$.



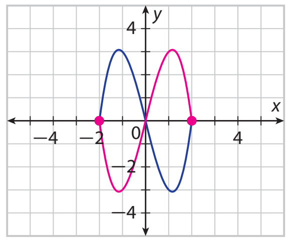
$g(x) = f(3x)$

8. Compress the graph of $f(x)$ vertically by a factor of $\frac{1}{2}$.



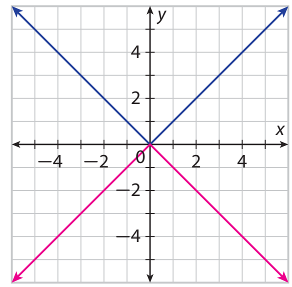
$g(x) = \frac{1}{2}f(x)$

9. Reflect the graph of $f(x)$ across the y-axis.



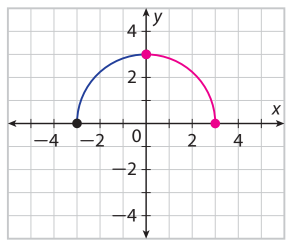
$g(x) = f(-x)$

10. Reflect the graph of $f(x)$ across the x-axis.



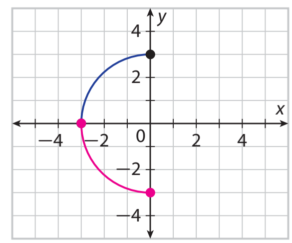
$g(x) = -f(x)$

11. Reflect the graph of $f(x)$ across the y-axis.



$g(x) = f(-x)$

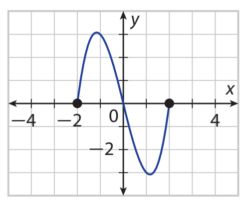
12. Reflect the graph of $f(x)$ across the x-axis.



$g(x) = -f(x)$

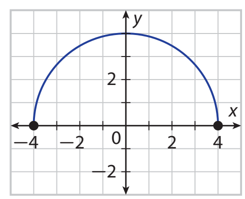
13. Determine if each function is an even function, an odd function, or neither.

a.



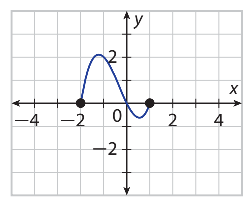
Odd function

b.



Even function

c.



Neither

Exercise	Depth of Knowledge (D.O.K.)	Math Processes and Practices
1–4	**1** Recall of Information	**MPP4** Mathematical Modeling
5–14	**1** Recall of Information	**MPP2** Abstract and Quantitative Reasoning
15–16	**1** Recall of Information	**MPP6** Using Precise Mathematical Language
17–18	**2** Skills/Concepts	**MPP4** Mathematical Modeling
19	**3** Strategic Thinking H.O.T.	**MPP2** Abstract and Quantitative Reasoning
20	**3** Strategic Thinking H.O.T.	**MPP6** Using Precise Mathematical Language

14. Determine whether each quadratic function is an even function. Answer *yes* or *no*.

a. $f(x) = 5x^2$ **Yes**

b. $f(x) = (x - 2)^2$ **No**

c. $f(x) = \left(\frac{x}{3}\right)^2$ **Yes**

d. $f(x) = x^2 + 6$ **Yes**

Describe how to transform the graph of $f(x) = x^2$ to obtain the graph of the related function $g(x)$. Then draw the graph of $g(x)$. **15–16. See margin.**

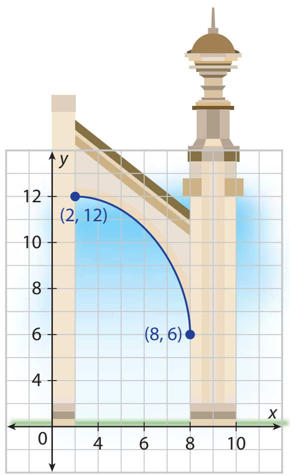
15. $g(x) = -\dfrac{f(x+4)}{3}$

16. $g(x) = f(2x) + 2$

17. Architecture Flying buttresses were used in the construction of cathedrals and other large stone buildings before the advent of more modern construction materials to prevent the walls of large, high-ceilinged rooms from collapsing.

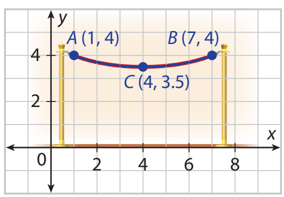
The design of a flying buttress includes an arch. In the illustration shown, the unit of measurement for both axes is feet, and the vertex of the arch is point C. Find a quadratic function that models the arch, and state the function's domain.

$g(x) = -\frac{1}{6}f(x-2) + 12$; Domain: $\{x \mid 2 \le x \le 8\}$



18. A red velvet rope hangs between two stanchions and forms a curve that can be modeled by a parabola. In the illustration shown, the unit of measurement for both axes is feet, and the vertex of the curve is point C. Find a quadratic function that models the rope, and state the function's domain.

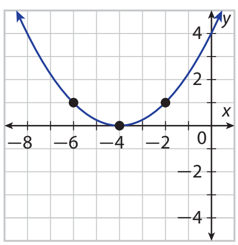
$g(x) = \frac{1}{18}f(x-4) + 3.5$; Domain: $\{x \mid 1 \le x \le 7\}$



H.O.T. Focus on Higher Order Thinking

19. Multiple Representations The graph of the function $g(x) = \left(\frac{1}{2}x + 2\right)^2$ is shown.

Use the graph to identify the transformations of the graph of $f(x) = x^2$ needed to produce the graph of $g(x)$. (If a stretch or compression is involved, give it in terms of a horizontal stretch or compression rather than a vertical one.) Use your list of transformations to write $g(x)$ in the form $g(x) = f\left(\frac{1}{b}(x-h)\right) + k$. Then show why the new form of $g(x)$ is algebraically equivalent to the given form. **See Additional Answers.**



Integrate Math Processes and Practices
Focus on Generalizing

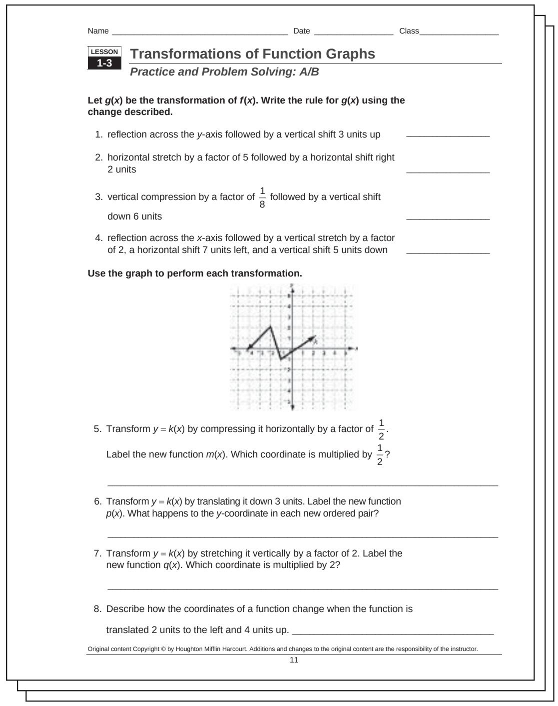
MPP8 Encourage students to use their graphing calculators to explore graphs of functions of higher degree and to see whether they can determine any patterns that would indicate when a function is even and when it is odd.

Focus on Abstract and Quantitative Reasoning

MPP2 Students can check their functions for correctness by substituting the coordinates of a point on the parabola into the rule and checking to see whether the resulting equation is true.

Online Resources

- Practice and Problem Solving (three forms)
- Reteach
- Reading Strategies
- Success for English Learners



Name ________ Date ________ Class ________

LESSON 1-3 **Transformations of Function Graphs**

Practice and Problem Solving: A/B

Let $g(x)$ be the transformation of $f(x)$. Write the rule for $g(x)$ using the change described.

1. reflection across the y-axis followed by a vertical shift 3 units up ________
2. horizontal stretch by a factor of 5 followed by a horizontal shift right 2 units ________
3. vertical compression by a factor of $\frac{1}{8}$ followed by a vertical shift down 6 units ________
4. reflection across the x-axis followed by a vertical stretch by a factor of 2, a horizontal shift 7 units left, and a vertical shift 5 units down ________

Use the graph to perform each transformation.

5. Transform $y = k(x)$ by compressing it horizontally by a factor of $\frac{1}{2}$. Label the new function $m(x)$. Which coordinate is multiplied by $\frac{1}{2}$?

____

6. Transform $y = k(x)$ by translating it down 3 units. Label the new function $p(x)$. What happens to the y-coordinate in each new ordered pair?

____

7. Transform $y = k(x)$ by stretching it vertically by a factor of 2. Label the new function $q(x)$. Which coordinate is multiplied by 2?

____

8. Describe how the coordinates of a function change when the function is translated 2 units to the left and 4 units up. ________

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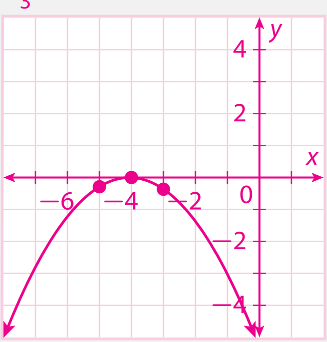
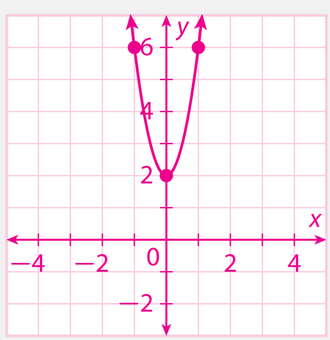
11

Answers

15. reflection of the graph of $f(x)$ across the x-axis, a vertical compression by a factor of $\frac{1}{3}$, and a translation of 4 units to the left



16. horizontal compression of the graph of $f(x)$ by a factor of $\frac{1}{2}$ and a translation of 2 units up



Journal

Have students explain how the parameters a, h, and k in the function $g(x) = af(x - h)^2 + k$ affect the graph of $f(x) = x^2$.

Connect Vocabulary EL

Some students may not be familiar with the term *armrest*, a compound word made up of *arm* and *rest*. Show students an example of a chair with armrests and another without armrests. Discuss how the armrests in the given design need to have the support of the parabolic legs beneath them to provide the strength to hold up under the weight of a person leaning on the armrests.

Questioning Strategies

Is a stretch or a compression used to form the legs of the chair with armrests? Is it vertical or horizontal? vertical stretch

Why is it necessary for the values of a to be negative in the equations for the parabolas? The parabolas open downward.

Which of the two chairs do you think is likely to be the more stable? Why? The chair with the armrests would be more stable because it is secured at two points on each pair of legs rather than just one point on each. The seat on the chair with the vertex centered on it could tip forward or backward depending on where the weight is shifted.

Answers

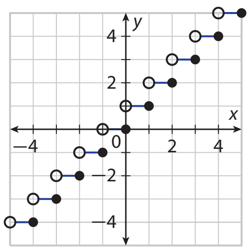
Sample answer: For the chair without armrests, $g(x) = -0.16f(x - 10) + 16$ with domain $\{x \mid 0 \leq x \leq 20\}$. For the chair with armrests, $h(x) = -0.24f(x - 10) + 24$, also with domain $\{x \mid 0 \leq x \leq 20\}$.

Lesson Performance Task Scoring Rubric

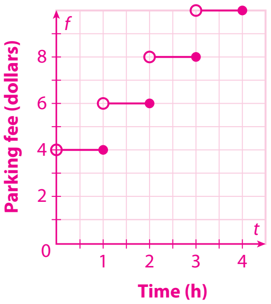
Points	Criteria
2	Student correctly solves the problem and explains his/her reasoning.
1	Student shows good understanding of the problem but does not fully solve or explain his/her reasoning.
0	Student does not demonstrate understanding of the problem.

20. **Represent Real-World Situations** The graph of the ceiling function, $f(x) = \lceil x \rceil$, is shown. This function accepts any real number x as input and delivers the least integer greater than or equal to x as output. For instance, $f(1.3) = 2$ because 2 is the least integer greater than or equal to 1.3. The ceiling function is a type of *step function*, so named because its graph looks like a set of steps.

Write a function $g(x)$ whose graph is a transformation of the graph of $f(x)$ based on this situation: A parking garage charges $4 for the first hour or less and $2 for every additional hour or fraction of an hour. Then graph $g(x)$.

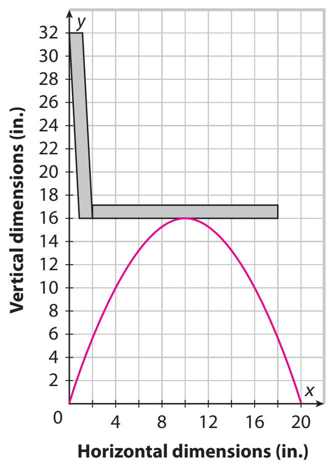


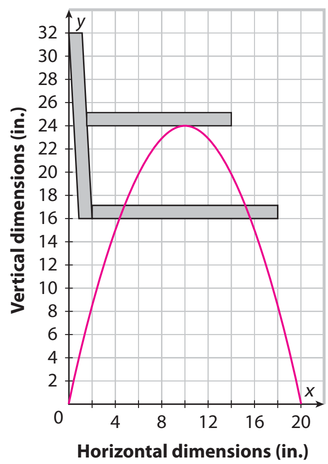
$g(x) = 2f(x) + 2$



Lesson Performance Task

You are designing two versions of a chair, one without armrests and one with armrests. The diagrams show side views of the chair. Rather than use traditional straight legs for your chair, you decide to use parabolic legs. Given the function $f(x) = x^2$, write two functions, $g(x)$ and $h(x)$, whose graphs represent the legs of the two chairs and involve transformations of the graph of $f(x)$. For the chair without armrests, the graph of $g(x)$ must touch the bottom of the chair's seat. For the chair with armrests, the graph of $h(x)$ must touch the bottom of the armrest. After writing each function, graph it. See margin for functions and domains.





Extension Activity

Have students research the differences between a catenary curve and a parabolic curve and find examples of catenaries in real life (for example, a hanging cable, or the Gateway Arch in St. Louis). Students should find that the two types of curves look very similar in that they are both symmetrical and have similar shapes. A parabola in its simplest form is $f(x) = x^2$ while a catenary is of the form $f(x) = \cosh(x)$. Parabolas are often used to model catenaries when the differences between them are not consequential.