1.1 Domain, Range, and End Behavior

Essential Question: How can you determine the domain, range, and end behavior of a function?

Resource Locker

Explore Representing an Interval on a Number Line

An **interval** is a part of a number line without any breaks. A *finite interval* has two endpoints, which may or may not be included in the interval. An *infinite interval* is unbounded at one or both ends.

Suppose an interval consists of all real numbers greater than or equal to 1. You can use the inequality $x \geq 1$ to represent the interval. You can also use *set notation* and *interval notation*, as shown in the table.

Description of Interval	Type of Interval	Inequality	Set Notation	Interval notation
All real numbers from a to b, including a and b	Finite	$a \leq x \leq b$	$\{x \mid a \leq x \leq b\}$	$[a, b]$
All real numbers greater than a	Infinite	$x > a$	$\{x \mid x > a\}$	$(a, +\infty)$
All real numbers less than or equal to a	Infinite	$x \leq a$	$\{x \mid x \leq a\}$	$(-\infty, a]$

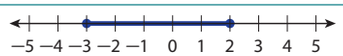
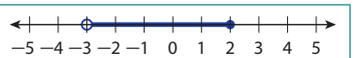
For set notation, the vertical bar means "such that," so you read $\{x \mid x \geq 1\}$ as "the set of real numbers x such that x is greater than or equal to 1."

For interval notation, do the following:

- Use a square bracket to indicate that an interval includes an endpoint and a parenthesis to indicate that an interval doesn't include an endpoint.
- For an interval that is unbounded at its positive end, use the symbol for positive infinity, $+\infty$. For an interval that unbounded at its negative end, use the symbol for negative infinity, $-\infty$. Always use a parenthesis with positive or negative infinity.

So, you can write the interval $x \geq 1$ as $[1, +\infty)$.

A Complete the table by writing the finite interval shown on each number line as an inequality, using set notation, and using interval notation.

Finite Interval		
Inequality	? $-3 \leq x \leq 2$	? $-3 < x \leq 2$
Set Notation	? $\{x \mid -3 \leq x \leq 2\}$	? $\{x \mid -3 < x \leq 2\}$
Interval Notation	? $[-3, 2]$	? $(-3, 2]$

Professional Development

Integrate Math Processes and Practices

This lesson provides an opportunity to address Math Process and Practice **MPP6,** which calls for students to use the language of mathematics to communicate precisely and to "attend to precision." Students learn to describe intervals using inequalities, set notation, and interval notation. They also learn how to use mathematical notation to describe end behavior of a function.

Domain, Range, and End Behavior

Learning Objective

Students will represent intervals on a number line, identify a function's domain, range, and end behavior from its graph, and graph a linear function on a restricted domain.

Math Processes and Practices

MPP6 Using Precise Mathematical Language

Language Objective

With a partner, fill in a graphic organizer showing the domain, range, and end behavior of a function.

Online Resources

An extra example for each Explain section is available online.

Engage

Essential Question: How can you determine the domain, range, and end behavior of a function?

Possible answer: The domain consists of x values for which the function is defined or on which the real-world situation is based. The range consists of the corresponding $f(x)$ values. The end behavior describes what happens to the $f(x)$ values as the x values increase without bound or decrease without bound.

Preview: Lesson Performance Task

View the online Engage. Discuss how the distance a car can travel is a function of the amount of gas in the car's gas tank. Have students identify the independent and dependent variables of the functional relationship. Then preview the Lesson Performance Task.

Explore

Representing an Interval on a Number Line

Integrate Technology

Students have the option of completing the activity either in the book or online.

Integrate Math Processes and Practices

Focus on Using Precise Mathematical Language

MPP6 Draw students' attention to the use of braces, parentheses, and brackets in the various representations. Make sure students can use the symbols correctly, and can explain the significance of the symbols in each type of notation.

Explain 1

Identifying a Function's Domain, Range, and End Behavior from its Graph

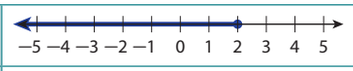
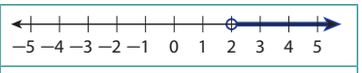
Avoid Common Errors

Some students may incorrectly identify the end behavior of a function that increases over the interval $(-\infty, 0)$ as "As $x \longrightarrow -\infty$, $f(x) \longrightarrow \infty$." Help students to see that for this part of the description, they must consider the behavior of the function as the values of *x decrease* (the behavior of the graph as observed from right to left), and not whether the function itself is an increasing or decreasing function.

Questioning Strategies

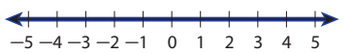
Is it possible that a linear function with the domain {all real numbers} could have a range that is *not* {all real numbers}? Explain. Yes; the function could be a constant function, such as $f(x) = 2$. The domain is {all real numbers}, but the range is {2}.

B Complete the table by writing the infinite interval shown on each number line as an inequality, using set notation, and using interval notation.

Infinite Interval		
Inequality	? $x \leq 2$	? $x > 2$
Set Notation	? $\{x \mid x \leq 2\}$	? $\{x \mid x > 2\}$
Interval Notation	? $(-\infty, 2]$	? $(2, +\infty)$

Reflect

1. Consider the interval shown on the number line.



a. Represent the interval using interval notation. $(-\infty, +\infty)$

b. What numbers are in this interval? **All real numbers**

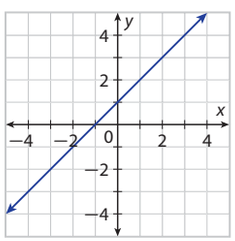
2. What do the intervals [0, 5], [0, 5), and (0, 5) have in common? What makes them different?

2. All three intervals contain all the numbers between 0 and 5. The interval [0, 5] also includes the endpoints 0 and 5, the interval [0, 5) includes only the endpoint 0, and the interval (0, 5) does not contain either endpoint.

3. **Discussion** The symbol ∪ represents the *union* of two sets. What do you think the notation $(-\infty, 0) \cup (0, +\infty)$ represents? **All real numbers except 0**

Explain 1 Identifying a Function's Domain, Range and End Behavior from its Graph

Recall that the *domain* of a function *f* is the set of input values *x*, and the *range* is the set of output values $f(x)$. The **end behavior** of a function describes what happens to the $f(x)$-values as the *x*-values either increase without bound (approach positive infinity) or decrease without bound (approach negative infinity). For instance, consider the graph of a linear function shown. From the graph, you can make the following observations.



Statement of End Behavior	Symbolic Form of Statement
As the *x*-values increase without bound, the *f*(*x*)-values also increase without bound.	As $x \longrightarrow +\infty$, $f(x) \longrightarrow +\infty$.
As the *x*-values decrease without bound, the *f*(*x*)-values also decrease without bound.	As $x \longrightarrow -\infty$, $f(x) \longrightarrow -\infty$.

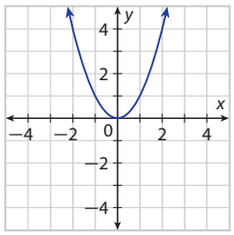
Collaborative Learning

Peer-to-Peer Activity

Have students work in pairs to draw the graph of a linear function with a restricted domain and keep the graph hidden from their partner. Have students describe the function's domain and range using one of the notation forms from the lesson, then exchange descriptions and try to draw each other's graph. Have students compare their graphs (which may differ) and discuss the results.

Example 1 **Write the domain and the range of the function as an inequality, using set notation, and using interval notation. Also describe the end behavior of the function.**

A The graph of the quadratic function $f(x) = x^2$ is shown.



Domain:

Inequality: $-\infty < x < +\infty$

Set notation: $\{x \mid -\infty < x < +\infty\}$

Interval notation: $(-\infty, +\infty)$

Range:

Inequality: $y \geq 0$

Set notation: $\{y \mid y \geq 0\}$

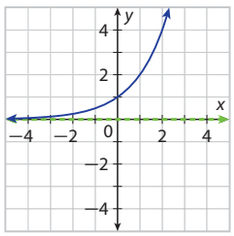
Interval notation: $[0, +\infty)$

End behavior:

As $x \to +\infty$, $f(x) \to +\infty$.

As $x \to -\infty$, $f(x) \to +\infty$.

B The graph of the exponential function $f(x) = 2^x$ is shown.



Domain:

Inequality: $-\infty < x < +\infty$

Set notation: $\{x \mid -\infty < x < +\infty\}$

Interval notation: $(-\infty, +\infty)$

Range:

Inequality: $y > 0$

Set notation: $\{y \mid y > 0\}$

Interval notation: $(0, +\infty)$

End behavior:

As $x \to +\infty$, $f(x) \to +\infty$.

As $x \to -\infty$, $f(x) \to 0$.

Reflect **Unlike the graph of a linear function, the graph of a quadratic function has a turning point (the vertex), which changes the direction of the graph.**

4. Why is the end behavior of a quadratic function different from the end behavior of a linear function?

5. In Part B, the $f(x)$-values decrease as the x-values decrease. So, why can't you say that $f(x) \to -\infty$ as $x \to -\infty$? **The *f(x)*-values do not decrease without bound. They instead approach 0.**

Explain 2

Graphing a Linear Function on a Restricted Domain

Integrate Math Processes and Practices
Focus on Problem Solving

MPP1 Remind students that the graph of a function represents the set of ordered pairs produced by the function. Help them to see that when they are using a graph to identify the range of a function, they are looking to identify the *y*-values of those ordered pairs.

Questioning Strategies

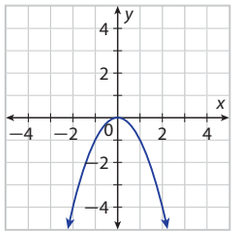
If a linear function has a restricted domain, must the range consist of a finite number of elements? Explain. No. If the domain is restricted to an interval (or intervals), as opposed to a finite number of elements, the range could consist of infinitely many values. For example, the range of the function $f(x) = 3x$ with domain $[0, 5]$ is $[0, 15]$, an interval containing infinitely many numbers.

If the domain of a linear function consists of n elements, how many elements would there be in the range? Explain. One, if the function is a constant function, or n if it is not. In a non-constant linear function, each element of the domain is paired with a different element of the range.

Your Turn

Write the domain and the range of the function as an inequality, using set notation, and using interval notation. Also describe the end behavior of the function.

6. The graph of the quadratic function $f(x) = -x^2$ is shown.



Domain: $-\infty < x < +\infty$, $\{x| -\infty < x < +\infty\}$, $(-\infty, +\infty)$

Range: $y \leq 0$, $\{y| y \leq 0\}$, $(-\infty, 0]$

End behavior: As $x \to +\infty$, $f(x) \to -\infty$; As $x \to -\infty$, $f(x) \to -\infty$.

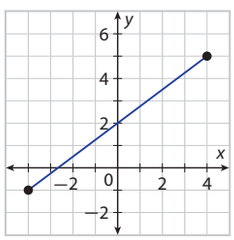
Explain 2 Graphing a Linear Function on a Restricted Domain

Unless otherwise stated, a function is assumed to have a domain consisting of all real numbers for which the function is defined. Many functions—such as linear, quadratic, and exponential functions—are defined all real numbers, so their domain, when written in interval notation, is $(-\infty, +\infty)$. Another way to write the set of real numbers is $\mathbb{R}$.

Sometimes a function may have a restricted domain. If the rule for a function and its restricted domain are given, you can draw its graph and then identify its range.

Example 2 **For the given function and domain, draw the graph and identify the range using the same notation as the given domain.**

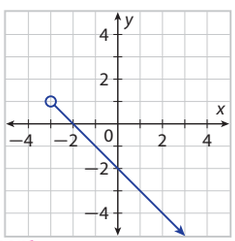
A $f(x) = \frac{3}{4}x + 2$ with domain $[-4, 4]$



Since $f(x) = \frac{3}{4}x + 2$ is a linear function, the graph is a line segment with endpoints at $(-4, f(-4))$, or $(-4, -1)$, and $(4, f(4))$, or $(4, 5)$. The endpoints are included in the graph.

The range is $[-1, 5]$.

B $f(x) = -x - 2$ with domain $\{x|x > -3\}$



Since $f(x) = -x - 2$ is a linear function, the graph is a ray with its endpoint at $(-3, f(-3))$,or $(-3, 1)$. The endpoint is not included in the graph. The range is $\{y|y < 1\}$.

Reflect

7. In Part A, how does the graph change if the domain is $(-4, 4)$ instead of $[-4, 4]$? **The graph no longer includes the endpoints of the segment.**

8. In Part B, what is the end behavior as x increases without bound? **As $x \to +\infty$, $f(x) \to -\infty$. Because** Why can't you talk about the end behavior as x decreases without bound? **the domain does not include values of x that are less than or equal to -3, the values of x cannot decrease without bound.**

Differentiate Instruction

Cognitive Strategies

If students have a difficult time consistently identifying the domain and range of functions, encourage them to use the phrase *depends on* instead of *is a function of*. For example, "The distance traveled by a car depends on the amount of gas in the tank." Help them to see that the elements of the range "depend on" the elements of the domain.

Your Turn

For the given function and domain, draw the graph and identify the range using the same notation as the given domain. **9–10. For graphs see margin.**

9. $f(x) = -\frac{1}{2}x + 2$ with domain $-6 \leq x < 2$

The range is $1 < y \leq 5$.

10. $f(x) = \frac{2}{3}x - 1$ with domain $(-\infty, 3]$

The range is $(-\infty, 1]$.

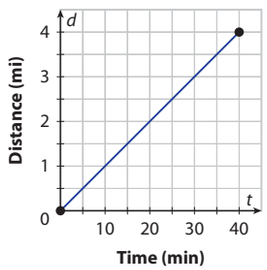
Explain 3 Modeling with a Linear Function

Recall that when a real-world situation involves a constant rate of change, a linear function is a reasonable model for the situation. The situation may require restricting the function's domain.

Example 3 **Write a function that models the given situation. Determine a domain from the situation, graph the function using that domain, and identify the range.**

(A) Joyce jogs at a rate of 1 mile every 10 minutes for a total of 40 minutes. (Use inequalities for the domain and range of the function that models this situation.)

Joyce's jogging rate is 0.1 mi/min. Her jogging distance d (in miles) at any time t (in minutes) is modeled by $d(t) = 0.1t$. Since she jogs for 40 minutes, the domain is restricted to the interval $0 \leq t \leq 40$.

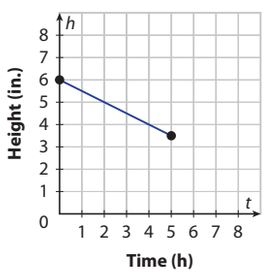


The range is $0 \leq d \leq 4$.

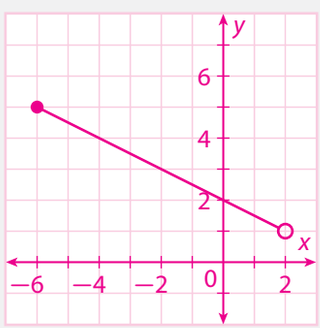
(B) A candle 6 inches high burns at a rate of 1 inch every 2 hours for 5 hours. (Use interval notation for the domain and range of the function that models this situation.)

The candle's burning rate is −0.5 in./h.
The candle's height h (in inches) at any time t (in hours) is modeled by $h(t) = 6 - 0.5t$. Since the candle burns for 5 hours, the domain is restricted to the interval $[0, 5]$.

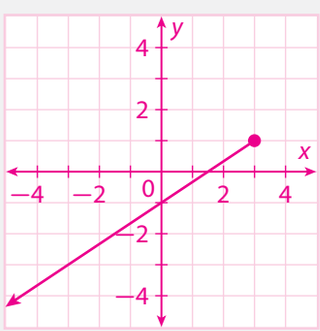
The range is $[3.5, 6]$.



9.



10.



Explain 3

Modeling with a Linear Function

Avoid Common Errors

Some students may erroneously identify the domain of a function that represents a real-world situation as an interval, when in fact the domain consists only of specific numbers *within* the interval, such as integers or multiples of a particular rational number. Help students to avoid this error by encouraging them to ask themselves whether any number within the interval could be a realistic input value for the situation described by the function.

Integrate Math Processes and Practices

Focus on Abstract and Quantitative Reasoning

MPP2 Encourage students to check their work by considering the reasonableness of the range of the function. Have them evaluate whether the values in the range seem realistic for the given situation.

Elaborate

Integrate Math Processes and Practices
Focus on Using and Evaluating Logical Reasoning

MPP3 Discuss different strategies for determining the range of a function from a graph of the function. Have students describe methods they use, and illustrate their methods using graphs of different types of functions, including those with restricted domains.

Questioning Strategies

Is it possible that a real-world situation can be modeled by a function whose domain consists of both positive and negative real numbers? If no, explain why not. If yes, give an example. Yes. Possible example: a function that describes the relationship between air temperature in degrees Fahrenheit and dew point, at a given level of humidity. The domain consists of all possible air temperatures in degrees Fahrenheit.

Summarize the Lesson

How do you identify and represent the domain, range, and end behavior of a function? To identify the domain, find the values of x for which the function is defined. To find the range, find the values of $f(x)$ for each value of x in the domain. There are different ways of representing the domain and range, including using inequalities, set notation, and interval notation. To find the end-behavior, consider what happens to the values of the range as the values of the domain increase or decrease without bound.

Reflect

11. In Part A, suppose Joyce jogs for only 30 minutes.

A. How does the domain change? **The domain is $0 \le t \le 30$ instead of $0 \le t \le 40$.**

B. How does the graph change? **The graph's right endpoint is (30, 3) instead of (40, 4).**

C. How does the range change? **The range is $0 \le d \le 3$ instead of $0 \le d \le 4$.**

Your Turn

12. While standing on a moving walkway at an airport, you are carried forward 25 feet every 15 seconds for 1 minute. Write a function that models this situation. Determine the domain from the situation, graph the function, and identify the range. Use set notation for the domain and range. **See Additional Answers.**

Elaborate **13–15. See Additional Answers.**

13. If a and b are real numbers such that $a < b$, use interval notation to write four different intervals having a and b as endpoints. Describe what numbers each interval includes.

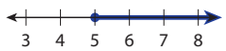
14. What impact does restricting the domain of a linear function have on the graph of the function?

15. **Essential Question Check-In** How does slope determine the end behavior of a linear function with an unrestricted domain?

Evaluate: Homework and Practice

Personal Math Trainer
- Online Homework
- Hints and Help
- Extra Practice

1. Write the interval shown on the number line as an inequality, using set notation, and using interval notation.



$x \ge 5$, $\{x|x \ge 5\}$, $[5, +\infty)$

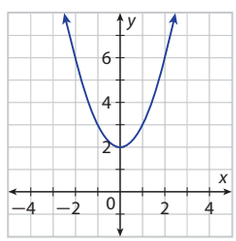
2. Write the interval (5, 100] as an inequality and using set notation.
$5 < x \le 100$; $\{x|5 < x \le 100\}$

3. Write the interval $-25 \le x < 30$ using set notation and interval notation.
$\{x| -25 \le x < 30\}$, $[-25, 30)$

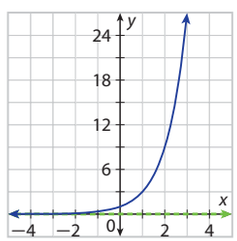
4. Write the interval $\{x| -3 < x < 5\}$ as an inequality and using interval notation.
$-3 < x < 5$, $(-3, 5)$

Write the domain and the range of the function as an inequality, using set notation, and using interval notation. Also describe the end behavior of the function or explain why there is no end behavior.

5. The graph of the quadratic function $f(x) = x^2 + 2$ is shown. **See above.**



6. The graph of the exponential function $f(x) = 3^x$ is shown.

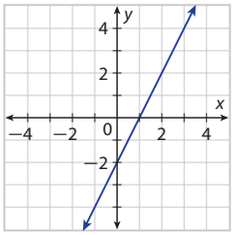


5. Domain:
$-\infty < x < +\infty$, $\{x| -\infty < x < +\infty\}$, $(-\infty, +\infty)$
Range: $2 \le y < +\infty$, $\{y|2 \le y < +\infty\}$, $[2, +\infty)$
End behavior: As $x \to +\infty$, $f(x) \to +\infty$; As $x \to -\infty$, $f(x) \to +\infty$.

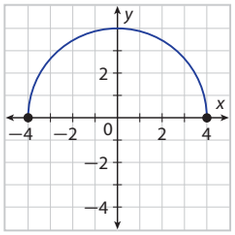
Domain:
$-\infty < x < +\infty$, $\{x| -\infty < x < +\infty\}$, $(-\infty, +\infty)$
Range: $0 < y < +\infty$, $\{y|0 < y < +\infty\}$, $(0, +\infty)$
End behavior: As $x \to +\infty$, $f(x) \to +\infty$; As $x \to -\infty$, $f(x) \to 0$.

Exercise	Depth of Knowledge (D.O.K.)	Math Processes and Practices
1–8	**1** Recall of Information	**MPP2** Abstract and Quantitative Reasoning
9–10	**1** Recall of Information	**MPP4** Mathematical Modeling
11	**1** Recall of Information	**MPP6** Using Precise Mathematical Language
12	**2** Skills/Concepts	**MPP4** Mathematical Modeling
13	**2** Skills/Concepts H.O.T.	**MPP3** Using and Evaluating Logical Reasoning
14	**3** Strategic Thinking H.O.T.	**MPP6** Using Precise Mathematical Language
15	**2** Skills/Concepts H.O.T.	**MPP4** Mathematical Modeling

7. The graph of the linear function $g(x) = 2x - 2$ is shown. **See right.**



8. The graph of a function is shown.



For the given function and domain, draw the graph and identify the range using the same notation as the given domain. **9–10. For graphs, see margin.**

9. $f(x) = -x + 5$ with domain $[-3, 2]$

The range is [3, 8].

10. $f(x) = \frac{3}{2}x + 1$ with domain $\{x \mid x > -2\}$

The range is $\{y \mid y > -2\}$

Write a function that models the given situation. Determine the domain from the situation, graph the function using that domain, and identify the range.

11. A bicyclist travels at a constant speed of 12 miles per hour for a total of 45 minutes. (Use set notation for the domain and range of the function that models this situation.) **For graph, see Additional Answers.**

$d(t) = 12t$

Domain: $\{t \mid 0 \le t \le 0.75\}$

Range: $\{d \mid 0 \le d \le 9\}$

12. An elevator in a tall building starts at a floor of the building that is 90 meters above the ground. The elevator descends 2 meters every 0.5 second for 6 seconds. (Use an inequality for the domain and range of the function that models this situation.) **For graph, see Additional Answers.**

$h(t) = 90 - 4t$; Domain: $0 \le t \le 6$; Range: $66 \le h \le 90$

H.O.T. Focus on Higher Order Thinking

13. Explain the Error Cameron sells tickets at a movie theater. On Friday night, she worked from 4 p.m. to 10 p.m. and sold about 25 tickets every hour. Cameron says that the number of tickets, n, she has sold at any time t (in hours) can be modeled by the function $n(t) = 25t$, where the domain is $0 \le t \le 1$ and the range is $0 \le n \le 25$. Is Cameron's function, along with the domain and range, correct? Explain.

Cameron's function is correct, but the domain and range are incorrect. Cameron worked for a total of 6 hours, so the domain of the function should be $0 \le t \le 6$. After 6 hours, Cameron has sold $25 \times 6 = 150$ tickets. So, the range of the function should be $0 \le n \le 150$.

7. Domain: $-\infty < x < +\infty$, $\{x \mid -\infty < x < +\infty\}$, $(-\infty, +\infty)$

Range: $-\infty < y < +\infty$, $\{y \mid -\infty < y < +\infty\}$, $(-\infty, +\infty)$

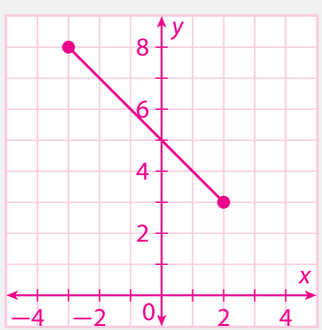
End behavior: As $x \to +\infty$, $f(x) \to +\infty$; As $x \to -\infty$, $f(x) \to -\infty$.

8. Domain: $-4 \le x \le 4$, $\{x \mid -4 \le x \le 4\}$, $[-4, 4]$

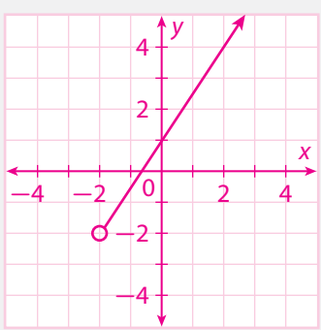
Range: $0 \le y \le 4$, $\{y \mid 0 \le y \le 4\}$, $[0, 4]$

There is no end behavior because the domain is bounded at both ends.

9.



10.



Evaluate

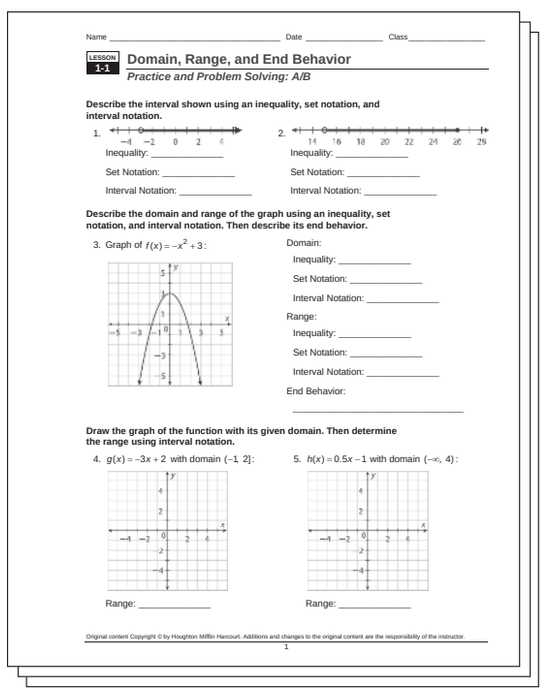
Assignment Guide

Level	Concepts and Skills	Practice
Basic	Explore	Exercises 1–4
	Example 1	Exercises 5 & 7
	Example 2	Exercise 9
	Example 3	Exercise 11
	H.O.T.	Exercise 14
Average	Explore	Exercises 1–4
	Example 1	Exercises 5–7
	Example 2	Exercises 9–10
	Example 3	Exercises 11–12
	H.O.T.	Exercises 13–14
Advanced	Explore	N/A
	Example 1	Exercises 6–8
	Example 2	Exercises 9–10
	Example 3	Exercises 11–12
	H.O.T.	Exercises 13–15

Real World Problems

Online Resources

- Practice and Problem Solving (three forms)
- Reteach
- Reading Strategies
- Success for English Learners



Name _____ Date _____ Class_____

LESSON 1-1 **Domain, Range, and End Behavior**

Practice and Problem Solving: A/B

Describe the interval shown using an inequality, set notation, and interval notation.

1. Inequality: _____ Set Notation: _____ Interval Notation: _____

2. Inequality: _____ Set Notation: _____ Interval Notation: _____

Describe the domain and range of the graph using an inequality, set notation, and interval notation. Then describe its end behavior.

3. Graph of $f(x) = -x^2 + 3$:

Domain: Inequality: _____ Set Notation: _____ Interval Notation: _____

Range: Inequality: _____ Set Notation: _____ Interval Notation: _____

End Behavior: _____

Draw the graph of the function with its given domain. Then determine the range using interval notation.

4. $g(x) = -3x + 2$ with domain $(-1, 2]$:

Range: _____

5. $h(x) = 0.5x - 1$ with domain $(-\infty, 4)$:

Range: _____

1

Journal

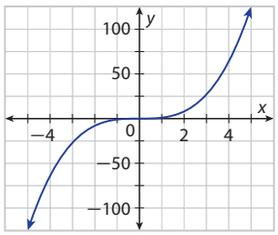
Have students describe how to identify the domain and range of a function given its graph, or given the situation it models.

Questioning Strategies

Why must restrictions be placed on the domain? The tank only holds at most 16 gallons of gas and at least 0 gallons of gas. A graph with a negative number of gallons of gas or more than 16 gallons of gas would not make sense for this situation.

Why does the domain contain all of the points in the interval and not just the integer values? The amount of gas used is continuous. There is a distance traveled for any value of g in the domain.

14. **Multi-Step** The graph of the cubic function $f(x) = x^3$ is shown.



14. See magin.

a. What are the domain, range, and end behavior of the function? (Write the domain and range as an inequality, using set notation, and using interval notation.)

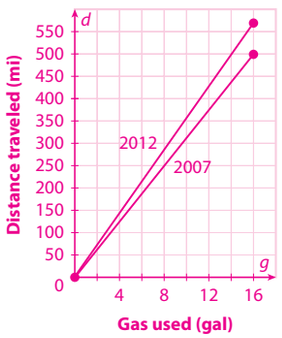
b. How is the range of the function affected if the domain is restricted to $[-4, 4]$? (Write the range as an inequality, using set notation, and using interval notation.)

c. Graph the function with the restricted domain.

15. **Represent Real-World Situations** The John James Audubon Bridge is a cable-stayed bridge in Louisiana that opened in 2011. The height from the bridge deck to the top of the tower where a particular cable is anchored is about 500 feet, and the length of that cable is about 1200 feet. Draw the cable on a coordinate plane, letting the x-axis represent the bridge deck and the y-axis represent the tower. (Only use positive values of x and y.) Write a linear function whose graph models the cable. Identify the domain and range, writing each as an inequality, using set notation, and using interval notation. **See Additional Answers.**

Lesson Performance Task

The fuel efficiency for a 2007 passenger car was 31.2 mi/gal. For the same model of car, the fuel efficiency increased to 35.6 mi/gal in 2012. The gas tank for this car holds 16 gallons of gas.



a. Write and graph a linear function that models the distance that each car can travel for a given amount of gas (up to one tankful).

b. Write the domain and range of each function using interval notation.

c. Write and simplify a function $f(g)$ that represents the *difference* in the distance that the 2012 car can travel and the distance that the 2007 car can travel on the same amount of gas. Interpret this function using the graphs of the functions from part a. Also find and interpret $f(16)$.

d. Write the domain and range of the difference function using set notation.

a. For both cars, let g be the amount of gas (in gallons) that each car uses, and let d be the distance (in miles) that each car travels. For the 2007 car, the linear model is $d_{2007}(g) = 31.2g$. For the 2012 car, the linear model is $d_{2012}(g) = 35.6g$.

b. The domain for the 2007 model is [0, 16], and the range is [0, 499.2]. The domain for the 2012 model is [0, 16], and the range is [0, 569.6].

c. The difference function is $f(g) = d_{2012}(g) - d_{2007}(g) = 35.6g - 31.2g = 4.4g$. This function gives the vertical distance between the graphs of $d_{2012}(g)$ and $d_{2007}(g)$. For instance, when $g = 16$, the vertical distance between the graphs is $f(16) = 4.4 \cdot 16 = 70.4$, which means the 2012 car can travel 70.4 miles farther on a tankful of gas than the 2007 car.

d. The domain is $\{g \mid 0 \le g \le 16\}$, and the range is $\{f(g) \mid 0 \le f(g) \le 70.4\}$.

Extension Activity

Have students research the average fuel costs per gallon in 2007 and in 2012. Have students use the data to create a new graph representing the distance each car could travel for a given amount of money (up to the cost of a typical full tank). Ask students to describe the aspects of their graphs.

Answers

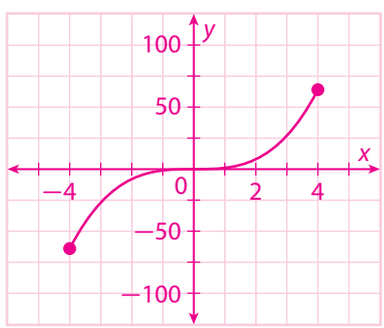
14. a. Domain:
$-\infty < x < +\infty$,
$\{x \mid -\infty < x < +\infty\}$,
$(-\infty, +\infty)$

Range: $-\infty < y < +\infty$,
$\{y \mid -\infty < y < +\infty\}$,
$(-\infty, +\infty)$

End behavior: As $x \to +\infty$, $f(x) \to +\infty$;
As $x \to -\infty$, $f(x) \to -\infty$.

b. Restricted range:
$-64 \le y \le 64$,
$\{y \mid -64 \le y \le +64\}$,
$[-64, 64]$

c.



Lesson Performance Task Scoring Rubric

Points	Criteria
2	Student correctly solves the problem and explains his/her reasoning.
1	Student shows good understanding of the problem but does not fully solve or explain his/her reasoning.
0	Student does not demonstrate understanding of the problem.