11.3 Solving Radical Equations

Essential Question: How can you solve equations involving square roots and cube roots?

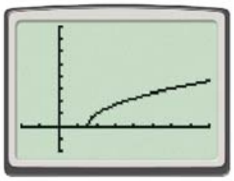
Resource Locker

Explore Investigating Solutions of Square Root Equations

When solving quadratic equations, you have learned that the number of real solutions depends upon the values in the equation, with different equations having 0, 1, or 2 real solutions. How many real solutions does a square root equation have? In the Explore, you will investigate graphically the numbers of real solutions for different square root equations.

(A) Remember that you can graph the two sides of an equation as separate functions to find solutions of the equation: a solution is any x-value where the two graphs intersect.

The graph of $y = \sqrt{x-3}$ is shown on a calculator window of $-4 \le x \le 16$ and $-2 \le y \le 8$. Reproduce the graph on your calculator. Then add the graph of $y = 2$.



How many solutions does the equation $\sqrt{x-3} = 2$ have? ? How do you know? ? **one; The graphs intersect at one point.**

On your calculator, replace the graph of $y = 2$ with the graph of $y = -1$.

How many solutions does the equation $\sqrt{x-3} = -1$ have? ? How do you know? ? **zero; The graphs never intersect.**

(B) Graph $y = \sqrt{x-3} + 2$ on your calculator (you can use the same viewing window as in Step A).

Add the graph of $y = 3$ to the graph of $y = \sqrt{x-3} + 2$.

How many solutions does $\sqrt{x-3} + 2 = 3$ have? ? **one**

Replace the graph of $y = 3$ with the graph of $y = 1$.

How many solutions does $\sqrt{x-3} + 2 = 1$ have? ? **zero**

Solving Radical Equations

Learning Objective

Students will solve simple equations involving radicals and rational exponents, identify extraneous solutions, and solve problems using radical equations.

Math Processes and Practices

MPP7 Seeing Structure

Language Objective

Work with a partner to complete a table to solve rational and radical equations.

Online Resources

An extra example for each Explain section is available online.

Engage

Essential Question: How can you solve equations involving square roots and cube roots?

First, combine terms to simplify, if possible. If there is one radical expression, isolate it on one side of the equation. Then, to solve the equation, square both sides when a square root is involved, and cube both sides when a cube root is involved. Finally, solve the resulting equation for the variable, remembering to check for extraneous solutions

Preview: Lesson Performance Task

View the Engage section online. Discuss the photo and how the strength of a tornado depends on its wind speed. Then preview the Lesson Performance Task.

Professional Development

Math Background

In this lesson, students use inverse operations to solve radical equations, which are equations in which the variable is in the radicand. For example, since the inverse of taking a cube root is raising to the third power, equations containing a cube root of a variable expression can be solved by cubing both sides of the equation. Squaring both sides of an equation may produce an extraneous solution; that is, a solution that is not a solution of the original equation.

Explore

Investigating Solutions of Square Root Equations

Integrate Technology

Students have the option of completing the Explore activity either in the book or online.

Questioning Strategies

How can the **INTERSECT** feature on your calculator help you solve a radical equation by graphing? You can graph the function represented by each side of the equation and find the x-value of the point of intersection, or determine that one does not exist.

Explain 1

Solving Square Root and $\frac{1}{2}$-Power Equations

Questioning Strategies

Why do you need to square both sides of the equation? Squaring is the inverse of taking a square root, so squaring the side that contains the radical gets rid of the radical sign. You have to square the other side, too, in order to maintain the equality.

Why don't all of the apparent solutions check? When you square both sides of an equation, you can introduce an extraneous, or false, solution.

Avoid Common Errors

When solving equations containing square roots, students may forget to square the entire quantity on each side. Caution students not to square terms individually when solving an equation such as $\sqrt{x+4} = x - 2$. Point out that when both sides of this equation are squared, the result is $x + 4 = (x-2)^2$, not $x + 4 = x^2 - 2^2$.

(C) Graph both sides of $\sqrt{4x-4} = x + 1$ as separate functions on your calculator.

How many solutions does $\sqrt{4x-4} = x + 1$ have? ? zero

Replace the graph of $y = x + 1$ with the graph of $y = \frac{1}{2}x$.

How many solutions does $\sqrt{4x-4} = \frac{1}{2}x$ have? ? two

Replace the graph of $y = \frac{1}{2}x$ with the graph of $y = 2x - 5$.

How many solutions does $\sqrt{4x-4} = 2x - 5$ have? ? one

(D) Graph both sides of $\sqrt{2x-3} = \sqrt{x}$ as separate functions on your calculator.

How many solutions does $\sqrt{2x-3} = \sqrt{x}$ have? ? one

Replace the graph of $y = \sqrt{x}$ with the graph of $y = \sqrt{2x+3}$.

How many solutions does $\sqrt{2x-3} = \sqrt{2x+3}$ have? ? zero

Reflect

1. When c is nonnegative, there is one solution. When c is negative, there are no solutions.

1. For a square root equation of the form $\sqrt{bx-h} = c$, what can you conclude about the number of solutions based on the sign of c?
2. For a square root equation of the form $\sqrt{bx-h} + k = c$, what can you conclude about the number of solutions based on the values of k and c? **See below.**
3. For a cube root equation of the form $\sqrt[3]{bx-h} = c$, will the number of solutions depend on the sign of c? Explain. **No, a cubic expression can have negative roots.**
4. The graphs in the second part of Step D appear to be get closer and closer as x increases. How can you be sure that they never meet, that is, that $\sqrt{2x-3} = \sqrt{2x+3}$ really has no solutions? **See below.**

Explain 1 Solving Square Root and $\frac{1}{2}$-Power Equations

A *radical equation* contains a variable within a radical or a variable raised to a (non-integer) rational power. To solve a square root equation, or, equivalently, an equation involving the power $\frac{1}{2}$, you can square both sides of the equation and solve the resulting equation.

Because opposite numbers have the same square, squaring both sides of an equation may introduce an apparent solution that is not an actual solution (an extraneous solution). For example, while the only solution of $x = 2$ is 2, the equation that is the square of each side, $x^2 = 4$, has two solutions, −2 and 2. But −2 is not a solution of the original equation.

2. When $k \le c$, there is one solution. When $k > c$, there are no solutions.

4. The graphs have the same shape. They are just different horizontal translations of $y = \sqrt{2x}$, one to the right and one to the left.

Collaborative Learning

Peer-to-Peer Activity

Have students work in pairs. Challenge each pair to write a radical equation that meets the following conditions.

- The solution of the equation is 6.
- The equation contains $2x$ on one side and $3x$ on the other.
- The radicand is the sum of a variable expression and a constant.

Have students then solve their equations, and identify any extraneous solutions. Have each pair share their work. Example: $\sqrt{2x+4} = 3x - 14$, extraneous: $\frac{32}{9}$

Example 1 **Solve the equation. Check for extraneous solutions.**

(A) $2 + \sqrt{x + 10} = x$

	$2 + \sqrt{x + 10} = x$
Isolate the radical.	$\sqrt{x + 10} = x - 2$
Square both sides.	$\left(\sqrt{x + 10}\right)^2 = (x - 2)^2$
Simplify.	$x + 10 = x^2 - 4x + 4$
Simplify.	$0 = x^2 - 5x - 6$
Factor.	$0 = (x - 6)(x + 1)$
Zero Product Property	$x = 6$ or $x = -1$

Check:

$2 + \sqrt{x + 10} = x$	$2 + \sqrt{x + 10} = x$
$2 + \sqrt{6 + 10} \stackrel{?}{=} 6$	$2 + \sqrt{-1 + 10} \stackrel{?}{=} -1$
$2 + \sqrt{16} \stackrel{?}{=} 6$	$2 + \sqrt{9} \stackrel{?}{=} -1$
$6 = 6$ ✓	$5 \neq -1$
$x = 6$ is a solution.	$x = -1$ is not a solution.

The solution is $x = 6$.

(B) $(x + 6)^{\frac{1}{2}} - (2x - 4)^{\frac{1}{2}} = 0$

Rewrite with radicals.	$\sqrt{x + 6} - \sqrt{2x - 4} = 0$
Isolate radicals on each side.	$\sqrt{x + 6} = \boxed{\sqrt{2x - 4}}$
Square both sides.	$\left(\sqrt{x + 6}\right)^2 = \left(\boxed{\sqrt{2x - 4}}\right)^2$
Simplify.	$\boxed{x + 6} = \boxed{2x - 4}$
Solve.	$\boxed{10} = x$

Check:

$$\sqrt{x + 6} - \sqrt{2x - 4} = 0$$

$$\sqrt{10 + 6} - \sqrt{2\left(\boxed{10}\right) - 4} \stackrel{?}{=} 0$$

$$\boxed{\sqrt{16}} - \boxed{\sqrt{16}} \stackrel{?}{=} 0$$

$$\boxed{0} = 0$$

The solution is $x = 10$.

Integrate Math Processes and Practices

Focus on Problem Solving

MPP1 Ask students to describe the relationship between the algebraic method used to solve a quadratic equation of the form $x^2 = a$ and the algebraic method used to solve a radical equation of the form $\sqrt{x} = b$. Lead them to see that the operations used to solve each equation are inverses of each other.

Integrate Technology

Students can use a spreadsheet to check their solutions. They can enter a possible solution in cell A1. In cell A2, they can enter the left side of the equation, using A1 as the variable. In cell B2, they can enter the right side of the equation, also using A1 as the variable. If the values in cells A2 and B2 are equal, the value in cell A1 is a solution.

Connect Vocabulary EL

Continue to make explicit the relationship between square roots, squaring, and raising to the power of 2; and cube roots, cubing, and raising to the power of 3. It is easy to overlook that what is familiar to native English speakers (the connection of *cube*, a 6-sided object, with the number 3, for example) may not be to English language learners.

Differentiate Instruction

Cognitive Strategies

Students have already solved many equations by performing the same operation on both sides to isolate the variable. They also have observed how inverse operations "undo" each other. Help them to see that they will be using this same idea to solve radical equations. The only difference is that they will be adding a new operation—raising both sides of an equation to a power—and that this operation may introduce extraneous solutions.

Explain 2

Solving Cube Root and $\frac{1}{3}$-Power Equations

Integrate Math Processes and Practices

Focus on Using and Evaluating Logical Reasoning

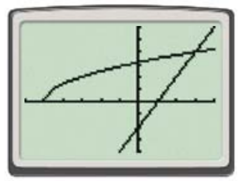
MPP3 Discuss with students why raising both sides of an equation to an even power may produce extraneous solutions, but raising to an odd power does not. Provide an opportunity for them to better understand *why* this is so by having them square and cube both sides of the equation $x = 5$, and consider solutions of the resulting equations.

Questioning Strategies

If an equation contains a variable expression raised to the one-third power, can the solution process produce extraneous solutions? How do you know? No; to solve the equation, you would need to cube both sides, and cubing both sides of an equation does not introduce the possibility of extraneous solutions.

Reflect

5. The graphs of each side of the equation from Part A are shown on the graphing calculator screen below. How can you tell from the graph that one of the two solutions you found algebraically is extraneous?



The graphs intersect at only 1 point, at the solution $x = 6$, so there is only one solution. The graphs do not intersect at the apparent solution $x = -1$.

Your Turn

6. Solve $(x + 5)^{\frac{1}{2}} - 2 = 1$. $x = 4$

Explain 2 Solving Cube Root and $\frac{1}{3}$-Power Equations

You can solve radical equations that involve roots other than square roots by raising both sides to the index of the radical. So, to solve a cube root equation, or, equivalently, an equation involving the power $\frac{1}{3}$, you can cube both sides of the equation and solve the resulting equation.

Example 2 **Solve the equation.**

(A) $\sqrt[3]{x+2} + 7 = 5$

$$\sqrt[3]{x+2} + 7 = 5$$

Isolate the radical. $\sqrt[3]{x+2} = -2$

Cube both sides. $\left(\sqrt[3]{x+2}\right)^3 = (-2)^3$

Simplify. $x + 2 = -8$

Solve for x $x = -10$

The solution is $x = -10$.

(B) $\sqrt[3]{x-5} = x + 1$

$$\sqrt[3]{x-5} = x + 1$$

Cube both sides. $\left(\sqrt[3]{x-5}\right)^3 = (x+1)^3$

Simplify $\boxed{x - 5} = \boxed{x^3 + 3x^2 + 3x + 1}$

Simplify. $0 = \boxed{x^3 + 3x^2 + 2x + 6}$

Begin to factor by grouping. $0 = x^2\left(\boxed{x+3}\right) + 2\left(\boxed{x+3}\right)$

Complete factoring $0 = (x^2 + 2)\left(\boxed{x+3}\right)$

Language Support EL

Communicate Math

Have students work in pairs to complete a table similar to the following, showing how to solve radical equations and equations with $\frac{1}{2}$ and $\frac{1}{3}$ power exponents. Have students write notes outlining the process next to each solution.

Type of equation	Example	Notes explaining steps
Radical: square root		
Equation with $\frac{1}{2}$ power exponent		
Radical: cube root		
Equation with $\frac{1}{3}$ power exponent		

By the Zero Product Property, $\boxed{x^2 + 2} = 0$ or $\boxed{x + 3} = 0$.

Because there are no real values of x for which $x^2 = \boxed{-2}$, the only solution is $\boxed{x = -3}$.

Reflect

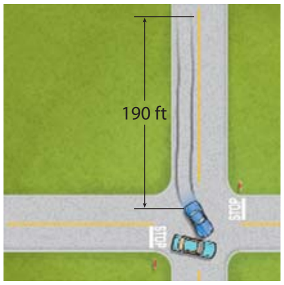
7. **Discussion** Example 1 shows checking for extraneous solutions, while Example 2 does not. While it is always wise to check your answers, can a cubic equation have an extraneous solution? Explain your answer. **See below.**

Your Turn

8. Solve $2(x - 50)^{\frac{1}{3}} = -10$. $\boldsymbol{x = -75}$

Explain 3 Solving a Real-World Problem

(A) **Driving** The speed s in miles per hour that a car is traveling when it goes into a skid can be estimated by using the formula $s = \sqrt{30fd}$, where f is the coefficient of friction and d is the length of the skid marks in feet.



After an accident, a driver claims to have been traveling the speed limit of 55 mi/h. The coefficient of friction under the conditions at the time of the accident was 0.6, and the length of the skid marks is 190 feet. Is the driver telling the truth about the car's speed? Explain.

Use the formula to find the length of a skid at a speed of 55 mi/h. Compare this distance to the actual skid length of 190 feet.

	$s = \sqrt{30fd}$
Substitute 55 for s and 0.6 for f	$55 = \sqrt{30(0.6)d}$
Simplify.	$55 = \sqrt{18d}$
Square both sides.	$55^2 = \left(\sqrt{18d}\right)^2$
Simplify.	$3025 = 18d$
Solve for d.	$168 \approx d$

If the driver had been traveling at 55 mi/h, the skid marks would measure about 168 feet. Because the skid marks actually measure 190 feet, the driver must have been driving faster than 55 mi/h.

7. No; while every positive number has two square roots, every number—positive or negative—has exactly one cube root. That is, the cube of every number is unique. So, when you cube both sides of an equation, you do not introduce the possibility of another number having the same cube as an actual solution.

Explain 3

Solving a Real-World Problem

Questioning Strategies

What are the restrictions on the variables in a square root function that models a real-world situation? The variables in the radicand are restricted to values that make the radicand non-negative. Also, the values of the variables are restricted to values that make sense in the given context.

Integrate Math Processes and Practices

Focus on Mathematical Modeling

MPP4 Discuss with students how each function models the given situation, explaining the relationship between the indicated variables. Help students to recognize that a model involving a square root relationship is the inverse of a model that involves a quadratic relationship.

Elaborate

Questioning Strategies

If the only apparent solution of a radical equation is an extraneous solution, what will be true about the graphs of the functions represented by the expressions on each side of the equation? Explain. The graphs will not intersect. The introduction of extraneous solutions is a consequence of algebraic manipulation. The solving of a radical equation by graphing produces only true solutions, represented by the *x*-coordinate of the point or points of intersection. If an equation has no true solution, the graphs will not intersect.

Summarize The Lesson

How do you solve an equation that contains a radical? First, isolate the radical. Then raise both sides of the equation to the power equal to the index of the radical. Finally, solve the resulting equation for the variable, and check for extraneous solutions.

Answers

11. There will always be one solution since there is a unique cube root for every real number. You should check your answer anyway, even though there won't be extraneous solutions, to make sure you haven't made any computational mistakes. Also, in a real-world context, you need to make sure the answer makes sense in the situation.
12. Squaring both sides; squaring both sides can create an apparent solution that is not a solution of the original equation, that is, an extraneous solution. This means that you must be sure to check for extraneous solutions.

(B) **Construction** The diameter d in inches of a rope needed to lift a weight of w tons is given by the formula $d = \frac{\sqrt{15w}}{\pi}$. How much weight can be lifted with a rope with a diameter of 1.0 inch?

Use the formula for the diameter as a function of weight, and solve for the weight given the diameter.

	$d = \frac{\sqrt{15w}}{\pi}$
Substitute.	$\boxed{1.0} = \frac{\sqrt{15w}}{\pi}$
Square both sides.	$\boxed{\pi} = \boxed{\sqrt{15w}}$
Isolate the radical.	$\left(\boxed{\pi}\right)^2 = \left(\sqrt{15w}\right)^2$
Simplify.	$\boxed{\pi^2} = 15w$
Solve for w.	$\boxed{0.66} \approx w$

A rope with a diameter of 1.0 can hold about 0.66 ton, or about 1320 pounds.

Your Turn

9. **Biology** The trunk length (in inches) of a male elephant can be modeled by $l = 23\sqrt[3]{t} + 17$, where t is the age of the elephant in years. If a male elephant has a trunk length of 100 inches, about what is his age?

The elephant is about 47 years old.

10. Isolating the radical gives $\sqrt{4x + 8} = -8$. Because the principal square root of a quantity cannot be negative, it is clear that there will be no real solution.

Elaborate

10. A student asked to solve the equation $\sqrt{4x + 8} + 9 = 1$ isolated the radical, squared both sides, and solved for x to obtain $x = 14$, only to find out that the apparent solution was extraneous. Why could the student have stopped trying to solve the equation after isolating the radical?
11. When you see a cube root equation with the radical expression isolated on one side and a constant on the other, what should you expect for the number of solutions? Explain. What are some reasons you should check your answer anyway? **See margin.**
12. **Essential Question Check-In** Solving a quadratic equation of the form $x^2 = a$ involves taking the square root of both sides. Solving a square root equation of the form $\sqrt{x} = b$ involves squaring both sides of the equation. Which of these operations can create an equation that is not equivalent to the original equation? Explain how this affects the solution process. **See margin.**

Evaluate: Homework and Practice

Personal Math Trainer
- Online Homework
- Hints and Help
- Extra Practice

Solve the equation.

1. $\sqrt{x-9}=5$ **$x=34$**
2. $\sqrt{3x}=6$ **$x=12$**
3. $\sqrt{x+3}=x+1$ **$x=1$**
4. $\sqrt{(15x+10)}=2x+3$ **$x=-\frac{1}{4}$ and $x=1$**
5. $(x+4)^{\frac{1}{2}}=6$ **$x=32$**
6. $(45-9x)^{\frac{1}{2}}=x-5$ **$x=5$**
7. $(x-6)^{\frac{1}{2}}=x-2$ **no real solutions**
8. $4(x-2)^{\frac{1}{2}}=(x+13)^{\frac{1}{2}}$ **$x=3$**
9. $5-\sqrt[3]{x-4}=2$ **$x=31$**
10. $2\sqrt[3]{3x+2}=\sqrt[3]{4x-9}$ **$x=-\frac{5}{4}$**
11. $\sqrt[3]{69x+35}=x+5$ **$x=-15$**
12. $\sqrt[3]{x+5}=x-1$ **$x=3$**
13. $(x+7)^{\frac{1}{3}}=(4x)^{\frac{1}{3}}$ **$x=\frac{7}{3}$**
14. $(5x+1)^{\frac{1}{4}}=4$ **$x=51$**
15. $(-44x-18)^{\frac{1}{3}}=-2x+3$ **$x=\frac{9}{2}$**
16. $2(x-1)^{\frac{1}{5}}=(2x-17)^{\frac{1}{5}}$ **$x=\frac{1}{2}$**

17. **Driving** The formula for the speed versus skid length in Example 3A assumes that all 4 wheel brakes are working at top efficiency. If this is not true, another parameter is included in the equation so that the equation becomes $s=\sqrt{30fdn}$ where n is the percent braking efficiency as a decimal. Accident investigators know that the rear brakes failed on a car, reducing its braking efficiency to 60%. On a dry road with a coefficient of friction of 0.7, the car skidded 250 feet. Was the car going above the speed limit of 60 mi/h when the skid began? **See below.**

18. **Anatomy** The surface area S of a human body in square meters can be approximated by $S=\sqrt{\frac{hm}{36}}$ where h is height in meters and m is mass in kilograms. A basketball player with a height of 2.1 meters has a surface area of about 2.7 m^2. What is the player's mass?

The player has a mass of about 125 kg.

17. Expected skid length under the conditions is 286 feet, and the actual skid distance was 250 feet, so the car was not exceeding the speed limit when the skid began.

Exercise	Depth of Knowledge (D.O.K.)	Math Processes and Practices
1–16	**1** Recall of Information	**MPP5** Using Mathematical Tools
17–20	**2** Skills/Concepts	**MPP4** Mathematical Modeling
21	**1** Recall of Information	**MPP5** Using Mathematical Tools
22–24	**2** Skills/Concepts H.O.T.	**MPP3** Using and Evaluating Logical Reasoning
25	**3** Strategic Thinking H.O.T.	**MPP3** Using and Evaluating Logical Reasoning

Evaluate

Personal Math Trainer

Assignment Guide

Level	Concepts and Skills	Practice
Basic	Explore	Exercise 21
	Example 1	Exercises 1–7
	Example 2	Exercises 9–14
	Example 3	Exercises 17, 20
	H.O.T.	Exercise 22–23
Average	Explore	Exercise 21
	Example 1	Exercises 2–7
	Example 2	Exercises 9–16
	Example 3	Exercises 18–19
	H.O.T.	Exercises 22–24
Advanced	Explore	Exercise 21
	Example 1	Exercises 3–8
	Example 2	Exercises 9–16
	Example 3	Exercises 17–20
	H.O.T.	Exercises 22–25

Real World Problems

Questioning Strategies

How do you know by which power you should raise both sides of the equation? If it's a radical equation, raise both sides to the same power as the index. If the equation has a rational exponent, rewrite it as a radical equation, then follow the rule above, or raise both sides to the reciprocal of the exponent.

How can you tell whether a solution to a radical equation is extraneous? If the solution doesn't satisfy the original radical equation, it is extraneous.

Visual Cues

Suggest that students circle or highlight the index of a radical equation so that they know to which power to raise each side of the equation when solving it. For square-root equations, have them write in the index of 2.

Communicating Math

Have students describe how it is possible for a solution found using valid algebraic steps to not be a solution of the original equation.

Questioning Strategies

How could you use a graphing calculator to solve an equation such as $\sqrt{3-2x}=5-\sqrt{1-x}$? You could enter the left side of the equation as **Y1** and the right side as **Y2**, graph the two functions, and find the *x*-coordinate of the point of intersection. You could also use the **TABLE** feature to find the *x*-value that produces the same *y*-value for the two functions.

Avoid Common Errors

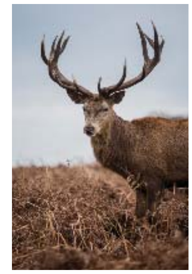
Some students believe that if a radical equation has two apparent solutions, one of them must be extraneous. To reinforce that this is not the case, have them solve the equation $\sqrt{5x-14}=x-2$ by graphing the functions defined by each side of the equation, and observing that their graphs intersect in two points. The solutions of the equation are the *x*-coordinates of these points, $x=3$ and $x=6$.

Integrate Math Processes and Practices

Focus on Abstract and Quantitative Reasoning

MPP2 Ask students to consider the equation $\sqrt{3x-5}=-2$, and how it is possible to know, without solving the equation, that it has no solution. Lead them to recognize that this is because the principal square root of a number is never negative; therefore, no value of x will satisfy the equation. Then have students solve the equation, both algebraically and graphically, and discuss their results.

19. **Biology** The approximate antler length L (in inches) of a deer buck can be modeled by $L=9\sqrt[3]{t}+15$ where t is the age in years of the buck. If a buck has an antler length of 36 inches, what is its age?



The buck is about 13 years old.

20. **Amusement Parks** For a spinning amusement park ride, the velocity v in meters per second of a car moving around a curve with radius r meters is given by $v=\sqrt{ar}$ where a is the car's acceleration in m/s^2. If the ride has a maximum acceleration of 30 m/s^2 and the cars on the ride have a maximum velocity of 12 m/s, what is the smallest radius that any curve on the ride may have?

The smallest radius that any curve on the ride may have is about 4.8 meters.

21. For each radical equation, state the number of solutions that exist.

A. $\sqrt{x-4}=-5$ **No solutions**

B. $\sqrt{x-4}+6=11$ **One solution**

C. $4=-2\sqrt[3]{x+2}$ **One solution**

D. $\sqrt{x+40}=0$ **One solution**

E. $\sqrt[3]{2x+5}=-18$ **One solution**

H.O.T. Focus on Higher Order Thinking

22. Critical Thinking For an equation of the form $\sqrt{x + a} = b$ where b is a constant, does the sign of a affect whether or not there is a solution for a given value of b? If so, how? If not, why not? **See below.**

23. Explain the Error Below is a student's work in solving the equation $2\sqrt{3x + 3} = 12$. What mistake did the student make? What is the correct solution?

$$2\sqrt{3x + 3} = 12$$
$$2\left(\sqrt{3x + 3}\right)^2 = 12^2$$
$$2(3x + 3) = 144$$
$$6x + 6 = 144$$
$$x = 23$$

When the student squared both sides, the coefficient 2 should also have been squared; $x = 11$

24. Communicate Mathematical Ideas Describe the key difference between solving radical equations for which you solve by raising both sides to an even power and those you solve by raising both sides to an odd power. **See below.**

25. Critical Thinking How could you solve an equation for which one side is a rational power expression and the other side is a constant? Give an example. Under what condition would you have to be especially careful to check your solutions?

Possible answer: One way is to raise both sides to the reciprocal power. For example, to solve $(2x - 3)^{\frac{3}{5}} = 8$, raise both sides to the $\frac{5}{3}$ power to obtain $2x - 3 = 32$, which has the solution $x = 17.5$. You would need to be especially careful when the denominator of the power is even, indicating an even root.

22. No; if you think of the graph of each side of the equation, the value of a just indicates a horizontal shift of the graph of the square root function and does not affect its range. So, it will not affect whether or not its graph and the graph of $y = b$ intersect.

24. Odd roots can be found for all values of the radicand, positive or negative, but even roots can only be found when the radicand is nonnegative. This means that raising both sides to an odd power will not introduce any extraneous solutions, since every cube is unique. But raising both sides to an even power may introduce extraneous solutions, since even powers of opposites are the same.

Critical Thinking

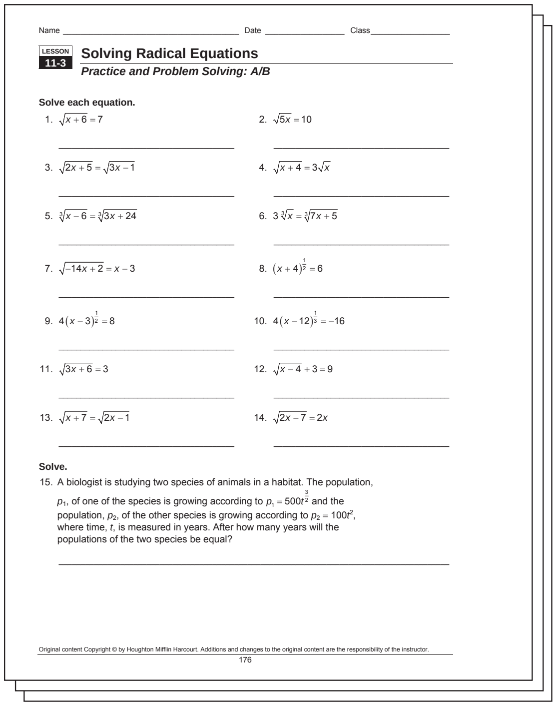
Challenge students to solve the equation $\sqrt[3]{2x} - \sqrt{3x} = 0$ algebraically. The equation can be solved by adding $\sqrt{3x}$ to both sides of the equation, cubing both sides of the resulting equation, and then squaring both sides of that resulting equation. The solutions are $x = 0$ and $x = \frac{4}{27}$.

Journal

Have students describe how inverse operations are used to solve radical equations.

Online Resources

- Practice and Problem Solving (three forms)
- Reteach
- Reading Strategies
- Success for English Learners



Name ______ Date ______ Class ______

LESSON 11-3 **Solving Radical Equations**
Practice and Problem Solving: A/B

Solve each equation.

1. $\sqrt{x + 6} = 7$
2. $\sqrt{5x} = 10$
3. $\sqrt{2x + 5} = \sqrt{3x - 1}$
4. $\sqrt{x + 4} = 3\sqrt{x}$
5. $\sqrt[3]{x - 6} = \sqrt[3]{3x + 24}$
6. $3\sqrt[3]{x} = \sqrt[3]{7x + 5}$
7. $\sqrt{-14x + 2} = x - 3$
8. $(x + 4)^{\frac{1}{2}} = 6$
9. $4(x - 3)^{\frac{1}{2}} = 8$
10. $4(x - 12)^{\frac{1}{3}} = -16$
11. $\sqrt{3x + 6} = 3$
12. $\sqrt{x - 4} + 3 = 9$
13. $\sqrt{x + 7} = \sqrt{2x - 1}$
14. $\sqrt{2x - 7} = 2x$

Solve.

15. A biologist is studying two species of animals in a habitat. The population, p_1, of one of the species is growing according to $p_1 = 500t^{\frac{3}{2}}$ and the population, p_2, of the other species is growing according to $p_2 = 100t^2$, where time, t, is measured in years. After how many years will the populations of the two species be equal?

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Avoid Common Errors

Some students may distribute the exponent to all terms in a set of parentheses. For example, students may want to write $V = k(F+2)^{\frac{3}{2}}$ as $V = k\left(F^{\frac{3}{2}} + 2^{\frac{3}{2}}\right)$ in order to isolate F. Instead, explain to students that they need to raise both sides of the equation to the reciprocal of the exponent:

$$V^{\frac{2}{3}} = \left(k(F+2)^{\frac{3}{2}}\right)^{\frac{2}{3}} = k^{\frac{2}{3}}(F+2)$$

Integrate Math Processes and Practices

Focus on Abstract and Quantitative Reasoning

MPP2 Have students discuss whether a linear wind scale would be more useful than a nonlinear one. Have students explain how they would compare tornadoes based on their Fujita numbers.

Have students discuss the challenges of designing a scale for tornado strength.

Lesson Performance Task Scoring Rubric

Points	Criteria
2	Student correctly solves the problem and explains his/her reasoning.
1	Student shows good understanding of the problem but does not fully solve or explain his/her reasoning.
0	Student does not demonstrate understanding of the problem.

Lesson Performance Task

For many years scientists have used a scale known as the Fujita Scale to categorize different types of tornados in relation to the velocity of the winds produced. The formula used to generate the scale is given by $V = k(F+2)^{\frac{3}{2}}$. The scale employs a constant, k, and the tornado's category number to determine wind speed. If you wanted to determine the different category numbers, how could you solve the radical equation for the variable F? (The value for k is about 14.1.) Solve the equation for F then verify the different categories using the minimum wind velocity. Do the values seem reasonable given the value for k?

Fujita Tornado Scale			
Damage Level	Category	Minimum Wind Velocity (mi/h)	Calculations
Moderate	F1	73	? $F \approx .9927$
Significant	F2	113	? $F \approx 2.0047$
Severe	F3	158	? $F \approx 3.0076$
Devastating	F4	207	? $F \approx 3.9956$
Incredible	F5	261	? $F \approx 4.9976$

$$F = \sqrt[3]{\frac{V^2}{14.1^2}} - 2$$

The values seem reasonable for the given value of *k*.

Extension Activity

Have students research the equation relating wind velocity to the Beaufort scale number, a measure of wind speed covering a broad range of phenomena, including tornadoes. Have students solve the equation for the Beaufort number, and then calculate the Beaufort number for the minimum wind velocities of each category of the Fujita scale. Have students discuss the relation between the Beaufort and Fujita scales.