11.1 Radical Expressions and Rational Exponents

Essential Question: How are rational exponents related to radicals and roots?

Resource Locker

Explore Defining Rational Exponents in Terms of Roots

Remember that a number a is an nth root of a number b if $a^n = b$. As you know, a square root is indicated by $\sqrt{}$ and a cube root by $\sqrt[3]{}$. In general, the n^{th} root of a real number a is indicated by $\sqrt[n]{a}$, where n is the **index** of the radical and a is the radicand. (Note that when a number has more than one real root, the radical sign indicates only the principal, or positive, root.)

A *rational exponent* is an exponent that can be expressed as $\frac{m}{n}$, where m is an integer and n is a natural number. You can use the definition of a root and properties of equality and exponents to explore how to express roots using rational exponents.

(A) How can you express a square root using an exponent? That is, if $\sqrt{a} = a^m$, what is m?

Given	$\sqrt{a} = a^m$
Square both sides.	$(\sqrt{a})^2 = (a^m)^2$
Definition of square root	$\boxed{a} = (a^m)^2$
Power of a power property	$a = a^{\boxed{2m}}$
Definition of first power	$a^{\boxed{1}} = a^{2m}$
The bases are the same, so equate exponents.	$\boxed{1} = \boxed{2m}$
Solve.	$m = \boxed{\frac{1}{2}}$
So,	$\sqrt{a} = a^{\boxed{\frac{1}{2}}}$

(B) How can you express a cube root using an exponent? That is, if $\sqrt[3]{a} = a^m$, what is m?

Given	$\sqrt[3]{a} = a^m$
Cube both sides.	$(\sqrt[3]{a})^3 = (a^m)^3$
Definition of cube root	$\boxed{a} = \boxed{(a^m)^3}$

Professional Development

Learning Progressions

In this lesson, students learn about rational exponents, and how to translate between radical expressions and expressions containing rational exponents. Students will use these skills in the next lesson, where they will learn how to apply the properties of rational exponents to simplify expressions containing radicals or rational exponents. They will also apply these skills to the solving of real-world problems that can be modeled by radical functions.

Radical Expressions and Rational Exponents

Learning Objective

Students will convert between radical expressions and expressions with rational exponents, and use power functions to model problem situations.

Math Processes and Practices

MPP6 Using Precise Mathematical Language

Language Objective

Identify, with a partner, matching radical expressions and rational equations.

Online Resources

An extra example for each Explain section is available online.

Engage

Essential Question: How are rational exponents related to radicals and roots?

Possible answer: Rational exponents and radicals are both ways to represent roots of quantities. The denominator of a rational exponent and the index of a radical represent the root. The rational exponent $\frac{m}{n}$ on a quantity represents the mth power of the nth root of the quantity or the nth root of the mth power of the quantity, where n is the index of the radical.

Preview: Lesson Performance Task

View the Engage section online. Discuss the photo and how both the air temperature and the wind speed can contribute to the wind chill. Then preview the Lesson Performance Task.

Explore

Defining Rational Exponents in Terms of Roots

Integrate Technology

Students have the option of completing the Explore activity either in the book or online.

Questioning Strategies

When rewriting a radical expression by using a rational exponent, where do you place the index of the radical? in the denominator of the rational exponent

How does knowing that $a^{\frac{1}{n}} = \sqrt[n]{a}$ help you to simplify the expression $16^{0.25}$? You can rewrite the fraction as $\frac{1}{4}$. So $16^{0.25} = 16^{\frac{1}{4}} = \sqrt[4]{16} = 2$.

Avoid Common Errors

Students may need to be reminded that although, for example, both 3 and –3 are fourth roots of 81, the expression $\sqrt[4]{81}$ indicates the *positive* (principal) fourth root of 81, or 3. Thus, the expression $81^{\frac{1}{4}}$ simplifies to 3, not to both 3 and –3.

Connect Vocabulary EL

In order to use accurate language in their explanations and questions, students need to understand the difference between the terms *radical* and *radicand*. Some students may not be familiar with the term *radicand.* Explain that it is *the number or expression under the radical sign*. When converting a radical expression to a power, the radicand becomes the base of the power.

Power of a power property	$a^{\boxed{?}} = \boxed{?}$	a^{3m}
Definition of first power	$a^1 \; \boxed{?} = \boxed{?}$	a^{3m}
The bases are the same, so equate exponents.	$1 \; \boxed{?} = \boxed{?}$	$3m$
Solve.	$m = \boxed{?}$	$\frac{1}{3}$
So,	$\sqrt[3]{a} = a^{\boxed{?}}$.	$\frac{1}{3}$

Reflect

1. **Discussion** Examine the reasoning in Steps A and B. Can you apply the same reasoning for any nth root, $\sqrt[n]{a}$, where n is a natural number? Explain. What can you conclude? **See below.**
2. For a positive number a, under what condition on n will there be only one real nth root? two real nth roots? Explain. **See Additional Answers.**
3. For a negative number a, under what condition on n will there be no real nth roots? one real nth root? Explain. **When n is even; when n is odd; no even power of any number is negative, but every number—positive or negative—has exactly one nth root when n is odd.**

Explain 1 Translating Between Radical Expressions and Rational Exponents

In the Explore, you found that a rational exponent $\frac{m}{n}$ with $m = 1$ represents an nth root, or that $a^{\frac{1}{n}} = \sqrt[n]{a}$ for positive values of a. This is also true for negative values of a when the index is odd. When $m \neq 1$, you can think of the numerator m as the power and the denominator n as the root. The following ways of expressing the exponent $\frac{m}{n}$ are equivalent.

Rational Exponents

For any natural number n, integer m, and real number a when the nth root of a is real:

Words	Numbers	Algebra
The exponent $\frac{m}{n}$ indicates the mth power of the nth root of a quantity.	$27^{\frac{2}{3}} = (\sqrt[3]{27})^2 = 3^2 = 9$	$a^{\frac{m}{n}} = (\sqrt[n]{a})^m$
The exponent $\frac{m}{n}$ indicates the nth root of the mth power of a quantity.	$4^{\frac{3}{2}} = \sqrt{4^3} = \sqrt{64} = 8$	$a^{\frac{m}{n}} = \sqrt[n]{a^m}$

Notice that you can evaluate each example in the "Numbers" column using the equivalent definition.

$$27^{\frac{2}{3}} = \sqrt[3]{27^2} = \sqrt[3]{729} = 9 \qquad 4^{\frac{3}{2}} = (\sqrt{4})^3 = 2^3 = 8$$

1. **Yes; the only difference is that instead of squaring or cubing both sides and using the definition of square root or cube root, you raise both sides to the nth power and use the definition of nth root. The other reasoning is exactly the same. You can conclude that finding the $\frac{1}{n}$ power is the same as finding the nth root, or that $\sqrt[n]{a} = a^{\frac{1}{n}}$.**

Collaborative Learning

Peer-to-Peer Activity

Have students work in pairs. Instruct each pair to create a quiz for another pair to take. Have them create five questions that involve simplifying a power that contains a rational exponent, and that contain expressions which can be simplified without the use of a calculator. Have pairs exchange quizzes with other pairs, work on the quiz with their partners, and check their answers with the pair that created the quiz.

Example 1 **Translate radical expressions into expressions with rational exponents, and vice versa. Simplify numerical expressions when possible. Assume all variables are positive.**

(A) **a.** $(-125)^{\frac{4}{3}}$ **b.** $x^{\frac{11}{8}}$ **c.** $\sqrt[5]{6^4}$ **d.** $\sqrt[4]{x^3}$

a. $(-125)^{\frac{4}{3}} = \left(\sqrt[3]{-125}\right)^4 = (-5)^4 = 625$

b. $x^{11/8} = \sqrt[8]{x^{11}}$ or $\left(\sqrt[8]{x}\right)^{11}$

c. $\sqrt[5]{6^4} = 6^{\frac{4}{5}}$

d. $\sqrt[4]{x^3} = x^{\frac{3}{4}}$

(B) **a.** $\left(\frac{81}{16}\right)^{\frac{3}{4}}$ **b.** $(xy)^{\frac{5}{3}}$ **c.** $\sqrt[3]{11^6}$ **d.** $\sqrt[3]{\left(\frac{2x}{y}\right)^5}$

a. $\left(\frac{81}{16}\right)^{\frac{3}{4}} = \left(\sqrt[4]{\frac{81}{16}}\right)^3 = \left(\boxed{\frac{3}{2}}\right)^3 = \boxed{\frac{27}{8}}$

b. $(xy)^{\frac{5}{3}} = \sqrt[3]{(xy)^5}$ or $\left(\sqrt[3]{xy}\right)^5$

c. $\sqrt[3]{11^6} = 11^{\boxed{\frac{6}{3}}} = 11^{\boxed{2}} = \boxed{121}$

d. $\sqrt[3]{\left(\frac{2x}{y}\right)^5} = \left(\frac{2x}{y}\right)^{\boxed{\frac{5}{3}}}$

Reflect

4. How can you use a calculator to show that evaluating $0.001728^{\frac{4}{3}}$ as a power of a root and as a root of a power are equivalent methods? **As a power of a root: Enter $\sqrt[3]{0.001728}$ to obtain 0.12. Then enter 0.12^4 to obtain 0.00020736. As a root of a power: Enter 0.001728^4. Then find $\sqrt[3]{(\text{Ans.})}$. The result is again 0.00020736.**

Your Turn

5. Translate radical expressions into expressions with rational exponents, and vice versa. Simplify numerical expressions when possible. Assume all variables are positive.

a. $\left(-\frac{32}{243}\right)^{\frac{2}{5}}$ $\mathbf{\frac{4}{9}}$

b. $(3y)^{\frac{b}{c}}$ $\left(\sqrt[c]{3y}\right)^b$ **or** $\sqrt[c]{(3y)^b}$

c. $\sqrt[3]{0.5^9}$ **0.125**

d. $\left(\sqrt[u]{st}\right)^v$ $(st)^{\frac{v}{u}}$

Explain 1

Translating Between Radical Expressions and Rational Exponents

Questioning Strategies

What does the numerator of a rational exponent indicate? the power of the expression What does the denominator indicate? the index of the radical

Integrate Math Processes and Practices

Focus on Using and Evaluating Logical Reasoning

MPP3 Discuss with students that when finding the value of a number raised to a rational exponent, they can evaluate either the root or the power first. Lead them to see that it is often easier to evaluate the root first, since doing so enables students to work with smaller numbers.

Differentiate Instruction

Cognitive Strategies

Students who continue to confuse the conversion of the numerator and denominator of the rational exponent to the exponent and index of the related radical expression may benefit from writing $\frac{e}{i}$ (e for exponent, i for index) next to the rational exponent before converting to the radical expression.

Explain 2

Modeling with Power Functions

Questioning Strategies

How is a power function related to a radical function? It is the same as the related radical function, just a way of expressing the function with a rational exponent instead of a radical.

How do you identify the restrictions on the domain of a power function that represents a real-world situation? The domain must be restricted to numbers that make x^b a real number, and further restricted to numbers that make sense in the context of the situation.

Integrate Technology

A graphing calculator can be used to explore the graph of the power function in the example. Students can also use the **TABLE** feature to identify the value of the function for different values of the domain.

Explain 2 Modeling with Power Functions

The following functions all involve a given power of a variable.

$A = \pi r^2$ (area of a circle)

$V = \frac{4}{3}\pi r^3$ (volume of a sphere)

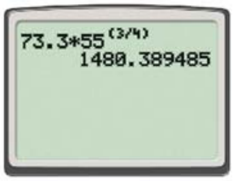
$T = 1.11 \cdot L^{\frac{1}{2}}$ (the time T in seconds for a pendulum of length L feet to complete one back-and-forth swing)

These are all examples of *power functions*. A power function has the form $y = ax^b$ where a is a real number and b is a rational number.

Example 2 **Solve each problem by modeling with power functions.**

(A) **Biology** The function $R = 73.3\sqrt[4]{M^3}$, known as Kleiber's law, relates the basal metabolic rate R in Calories per day burned and the body mass M of a mammal in kilograms. The table shows typical body masses for some members of the cat family.

Typical Body Mass	
Animal	**Mass (kg)**
House cat	4.5
Cheetah	55
Lion	170



a. Rewrite the formula with a rational exponent.

b. What is the value of R for a cheetah to the nearest 50 Calories?

c. From the table, the mass of the lion is about 38 times that of the house cat. Is the lion's metabolic rate more or less than 38 times the cat's rate? Explain.

a. Because $\sqrt[n]{a^m} = a^{\frac{m}{n}}$, $\sqrt[4]{M^3} = M^{\frac{3}{4}}$, so the formula is $R = 73.3M^{\frac{3}{4}}$.

b. Substitute 55 for M in the formula and use a calculator.

The cheetah's metabolic rate is about 1500 Calories.

c. Less; find the ratio of R for the lion to R for the house cat.

$$\frac{73.3(170)^{\frac{3}{4}}}{73.3(4.5)^{\frac{3}{4}}} = \frac{170^{\frac{3}{4}}}{4.5^{\frac{3}{4}}} \approx \frac{47.1}{3.1} \approx 15$$

The metabolic rate for the lion is only about 15 times that of the house cat.

(B) The function $h(m) = 241m^{-\frac{1}{4}}$ models an animal's approximate resting heart rate h in beats per minute given its mass m in kilograms.

a. A common shrew has a mass of only about 0.01 kg. To the nearest 10, what is the model's estimate for this shrew's resting heart rate?

b. What is the model's estimate for the resting heart rate of an American elk with a mass of 300 kg?

Language Support EL

Visual Cues

Have students work in pairs. Provide each pair with index cards on which are written either rational expressions or matching radical expressions. Include some radical expressions with square roots, rational expressions with rational exponents with a numerator of 1, and so on. Have students match cards and use colors or shapes to circle matching powers and indices. Suggest they write a 2 as an index of a square root, and a 1 as a power, to show an appropriate match for the special cases mentioned above.

c. Two animal species differ in mass by a multiple of 10. According to the model, about what percent of the smaller animal's resting heart rate would you expect the larger animal's resting heart rate to be?

a. Substitute 0.01 for m in the formula and use a calculator.

$$h(m) = 241\left(\boxed{0.01}\right)^{-\frac{1}{4}} \approx \boxed{760}$$

The model estimates the shrew's resting heart rate to be about 760 beats per minute.

b. Substitute 300 for m in the formula and use a calculator.

$$h(m) = 241\left(\boxed{300}\right)^{-\frac{1}{4}} \approx \boxed{60}$$

The model estimates the elk's resting heart rate to be about 60 beats per minute.

c. Find the ratio of $h(m)$ for the larger animal to the smaller animal. Let 1 represent the mass of the smaller animal.

$$\frac{241 \cdot \boxed{10}^{-\frac{1}{4}}}{241 \cdot 1^{-\frac{1}{4}}} = \boxed{10}^{-\frac{1}{4}} = \frac{1}{10^{\boxed{\frac{1}{4}}}} \approx \boxed{0.56}$$

You would expect the larger animal's resting heart rate to be about 56% of the smaller animal's resting heart rate.

Reflect

6. A power function involves a given power of a variable, while an exponential function involves a variable power of a given number (the base).

6. What is the difference between a power function and an exponential function?

7. In Part B, the exponent is negative. Are the results consistent with the meaning of a negative exponent that you learned for integers? Explain. **See margin.**

Your Turn

8. Use Kleiber's law from Part A.

a. Find the basal metabolic rate for a 170 kilogram lion to the nearest 50 Calories. Then find the formula's prediction for a 70 kilogram human. **for lion: 3450 Calories; for human: 1750 Calories**

b. Use your metabolic rate result for the lion to find what the basal metabolic rate for a 70 kilogram human *would* be *if* metabolic rate and mass were directly proportional. Compare the result to the result from Part a. **See margin.**

Elaborate

9. You can first rewrite the decimal as the fraction $\frac{25}{10} = \frac{5}{2}$. Then $4^{2.5} = 4^{\frac{5}{2}} = \left(\sqrt{4}\right)^5 = 2^5 = 32$.

9. Explain how can you use a radical to write and evaluate the power $4^{2.5}$.

10. When $y = kx$ for some constant k, y varies directly as x. When $y = kx^2$, y varies directly as the square of x; and when $y = k\sqrt{x}$, y varies directly as the square root of x. How could you express the relationship $y = kx^{\frac{3}{5}}$ for a constant k? ***y* varies directly as the three-fifths power of *x*.**

Answer

8b. If metabolic rate and mass were directly proportional then the metabolic rate would be about 1400 Cal. So, the rate for a human would be significantly lower than the actual prediction from Kleiber's law. Kleiber's law indicates that smaller organisms have a higher metabolic rate per kilogram of mass than do larger organisms.

Connect Vocabulary EL

For English language learners, differentiating between the words *rational* and *radical* can be difficult, both in print and in speech. Continue to make explicit connections between the terms' meanings and symbols each time they are used.

Elaborate

Integrate Math Processes and Practices

Focus on Abstract and Quantitative Reasoning

MPP2 Ask students to consider how they can use the fact that $a^{\frac{m}{n}} = \sqrt[n]{a^m}$ to prove that $\sqrt[k]{5^k} = 5$, given that k is any positive integer. Then ask them to create some examples using other bases and different values of k to verify this identity.

Summarize The Lesson

How can you rewrite a radical expression as an exponential expression and vice versa? You can write a radical expression as the radicand raised to a fraction in which the numerator is the power of the radicand and the denominator is the index of the radical. You can write an exponential expression with the base of the exponent as the radicand, the denominator of the exponent as the index, and the numerator of the exponent as the power.

Answer

7. Yes; a power with a negative integer exponent is the reciprocal of the corresponding positive power. So, for example, for the elk, this would mean that $300^{-\frac{1}{4}} = \frac{1}{300^{\frac{1}{4}}}$. Using the calculator again, $h(m) = 241\left(\frac{1}{300^{\frac{1}{4}}}\right) \approx 60$, which is consistent.

Evaluate

Assignment Guide

Level	Concepts and Skills	Practice
Basic	Explore	N/A
	Example 1	Exercises 1–16
	Example 2	Exercise 17
	H.O.T.	Exercise 22
Average	Explore	N/A
	Example 1	Exercises 1–16
	Example 2	Exercises 17–18
	H.O.T.	Exercises 22–23
Advanced	Explore	N/A
	Example 1	Exercises 5–18, 13–16
	Example 2	Exercises 18–20
	H.O.T.	Exercises 22–24

Real World Problems

Questioning Strategies

If the radicand is a negative number, what must be true about the index? Explain. The index must be odd. This is because even roots of negative numbers are not real numbers. (You can't raise a real number to an even power and get a negative number.)

Visual Cues

Suggest that students circle the denominator in the rational exponent, and draw a curved arrow from the denominator, passing beneath the base, to a point in front of the expression, indicating that it becomes the index of the radical in the converted expression.

Avoid Common Errors

When using a calculator to evaluate an expression that contains a rational exponent, students often forget to put parentheses around the exponent. Use an example, such as $8^{\frac{1}{3}}$, which students can simplify mentally, to show that the value of the expression when entered without parentheses is not the same as the value of the expression when entered correctly.

11. Essential Question Check-In Which of the following are true? Explain.

- To evaluate an expression of the form $a^{\frac{m}{n}}$, first find the nth root of a. Then raise the result to the mth power.
- To evaluate an expression of the form $a^{\frac{m}{n}}$, first find the mth power of a. Then find the nth root of the result. **They are both true. For a real number a and integers m and n with $n \neq 0$, $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$, so the order in which you find the root or power does not matter.**

Evaluate: Homework and Practice

- Online Homework
- Hints and Help
- Extra Practice

Translate expressions with rational exponents into radical expressions. Simplify numerical expressions when possible. Assume all variables are positive.

1. $64^{\frac{5}{3}}$ **1024**

2. $x^{\frac{p}{q}}$ $\sqrt[q]{x^p}$ **or** $\left(\sqrt[q]{x}\right)^p$

3. $(-512)^{\frac{2}{3}}$ **64**

4. $3^{\frac{2}{7}}$ $\sqrt[7]{9}$

5. $-\left(\frac{729}{64}\right)^{\frac{5}{6}}$ $-\frac{243}{32}$

6. $0.125^{\frac{4}{3}}$ **0.0625**

7. $vw^{\frac{2}{3}}$ $v\sqrt[3]{w^2}$ **or** $v\left(\sqrt[3]{w}\right)^2$

8. $(-32)^{0.6}$ **−8**

Translate radical expressions into expressions with rational exponents. Simplify numerical expressions when possible. Assume all variables are positive.

9. $\sqrt[7]{y^5}$ $y^{\frac{5}{7}}$

10. $\sqrt[7]{(-6)^6}$ $(-6)^{\frac{6}{7}}$

11. $\sqrt[3]{3^{15}}$ **243**

12. $\sqrt[4]{(\pi z)^3}$ $(\pi z)^{\frac{3}{4}}$

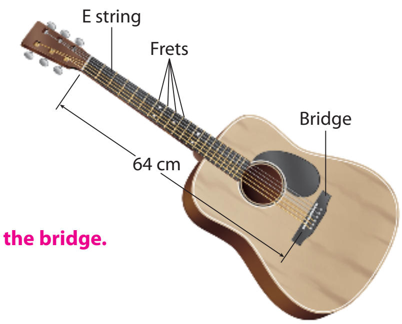
13. $\sqrt[6]{(bcd)^4}$ $(bcd)^{\frac{2}{3}}$

14. $\sqrt{6^6}$ **216**

15. $\sqrt[5]{32^2}$ **4**

16. $\sqrt[3]{\left(\frac{4}{x}\right)^9}$ $\frac{64}{x^3}$

17. Music Frets are small metal bars positioned across the neck of a guitar so that the guitar can produce the notes of a specific scale. To find the distance a fret should be placed from the bridge, multiply the length of the string by $2^{-\frac{n}{12}}$, where n is the number of notes higher than the string's root note. Where should a fret be placed to produce a F note on a B string (6 notes higher) given that the length of the string is 64 cm?



The fret should be placed about 45.25 cm from the bridge.

Exercise	Depth of Knowledge (D.O.K.)	Math Processes and Practices
1–16	**1** Recall of Information	**MPP5** Using Mathematical Tools
17	**2** Skills/Concepts	**MPP2** Abstract and Quantitative Reasoning
18–20	**2** Skills/Concepts	**MPP4** Mathematical Modeling
21	**1** Recall of Information	**MPP5** Using Mathematical Tools
22	**3** Strategic Thinking H.O.T.	**MPP2** Abstract and Quantitative Reasoning
23–24	**3** Strategic Thinking H.O.T.	**MPP3** Using and Evaluating Logical Reasoning

18. **Meteorology** The function $W = 35.74 + 0.6215T - 35.75V^{\frac{4}{25}} + 0.4275TV^{\frac{4}{25}}$ relates the windchill temperature W to the air temperature T in degrees Fahrenheit and the wind speed V in miles per hour. Use a calculator to find the wind chill temperature to the nearest degree when the air temperature is 28 °F and the wind speed is 35 miles per hour.

The windchill temperature is about 11°F.

19. **Astronomy** New stars can form inside a cloud of interstellar gas when a cloud fragment, or *clump*, has a mass M greater than the *Jean's mass* M_J. The Jean's mass is $M_J = 100n^{-\frac{1}{2}}(T + 273)^{\frac{3}{2}}$ where n is the number of gas molecules per cubic centimeter and T is the gas temperature in degrees Celsius. A gas clump has $M = 137$, $n = 1000$, and $T = -263$. Will the clump form a star? Justify your answer.

Yes; for this n and T, the Jean's mass is $M_J = 100(1000)^{-\frac{1}{2}}(-263 + 273)^{\frac{3}{2}} = \frac{100}{1000^{\frac{1}{2}}}(10)^{\frac{3}{2}} = 100.$ The mass of the clump, 137, is greater than the Jean's mass, 100, so the clump will form a star.

20. **Urban geography** The total wages W in a metropolitan area compared to its total population p can be approximated by a power function of the form $W = a \cdot p^{\frac{9}{8}}$ where a is a constant. About how many times as great does the model predict the total earnings for a metropolitan area with 3,000,000 people will be as compared to a metropolitan area with a population of 750,000?

about 4.8 times as great

21. Which statement is true? **C**

A. In the expression $8x^{\frac{3}{4}}$, $8x$ is the radicand.

B. In the expression $(-16)x^{\frac{4}{5}}$, 4 is the index.

C. The expression $1024^{\frac{n}{m}}$ represents the mth root of the nth power of 1024.

D. $50^{-\frac{2}{5}} = -50^{\frac{2}{5}}$

E. $\sqrt{(xy)^3} = xy^{\frac{3}{2}}$

H.O.T. Focus on Higher Order Thinking

22. **Explain the Error** A teacher asked students to evaluate $10^{-\frac{3}{5}}$ using their graphing calculators. The calculator entries of several students are shown below. Which entry will give the incorrect result? Explain.

$10^{(-3/5)}$ $\sqrt[5]{10^{-3}}$ 10^{-6} $1/10^{5/3}$ $\left(1/10^{1/5}\right)^3$

$\frac{1}{10^{\frac{5}{3}}}$; the negative exponent means to take the reciprocal of the corresponding positive power. The corresponding positive power is $\frac{3}{5}$, so the correct entry is $\frac{1}{10^{\frac{3}{5}}}$.

Integrate Technology

Students can use a graphing calculator to check their work. The **MATH** submenu, in the **MATH** menu, contains a cube root function as well as a function that can be used for radicals with other indices.

Integrate Math Processes and Practices

Focus on Using Precise Mathematical Language

MPP6 To help solidify students' understanding, have them verbalize their solutions to the exercises using accurate language. For a problem involving the simplification of , $32^{\frac{3}{5}}$ for example, a student might describe the solution in this way: "Thirty-two raised to the three-fifths power is equal to the fifth root of thirty-two raised to the third power, which is equal to two raised to the third power, which is equal to eight."

Peer-to-Peer Discussion

Ask students to work with a partner to determine two expressions of the form , $a^{\frac{m}{n}}$ where $\frac{m}{n}$ is not an integer, that are equal in value. Have students share their examples with the class and look for commonalities. Possible answers: $4^{\frac{5}{2}}$ and $8^{\frac{5}{3}}$, $27^{\frac{2}{3}}$ and $81^{\frac{1}{2}}$

Journal

Have students write two different representations, one as a radical and the other as a power, of the principal fourth root of the cube of 81. Then have them describe how they would find this value.

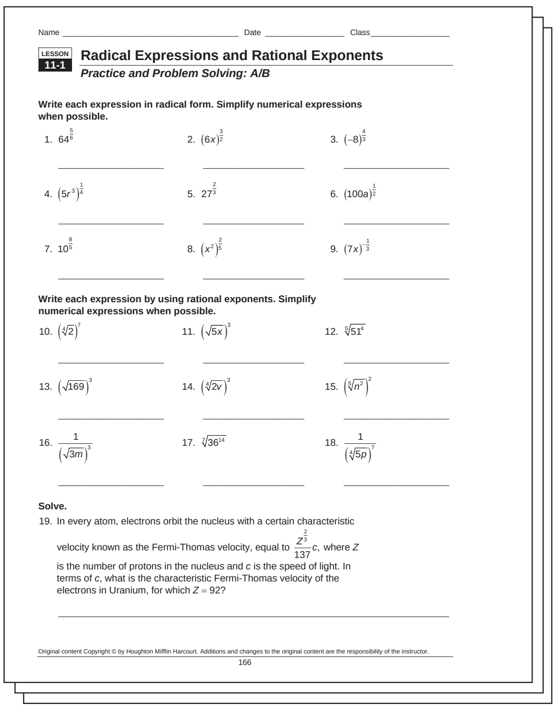
Avoid Common Errors

When solving the equation for V, some students may raise each *term* in the equation to the power $\frac{25}{4}$. Ask students what they need to do first. Isolate the variable V on one side of the equation.

Mention that after the equation is in the form $V^{\frac{4}{25}} = \frac{37.6}{18.65}$, each *side* of the equation can be raised to the power $\frac{25}{4}$.

Online Resources

- Practice and Problem Solving (three forms)
- Reteach
- Reading Strategies
- Success for English Learners



Name ______ Date ______ Class ______

LESSON 11-1 **Radical Expressions and Rational Exponents**

Practice and Problem Solving: A/B

Write each expression in radical form. Simplify numerical expressions when possible.

1. $64^{\frac{5}{6}}$
2. $(6x)^{\frac{3}{2}}$
3. $(-8)^{\frac{4}{3}}$
4. $(5r^3)^{\frac{1}{4}}$
5. $27^{\frac{2}{3}}$
6. $(100a)^{\frac{1}{2}}$
7. $10^{\frac{8}{5}}$
8. $(x^2)^{\frac{2}{5}}$
9. $(7x)^{-\frac{1}{3}}$

Write each expression by using rational exponents. Simplify numerical expressions when possible.

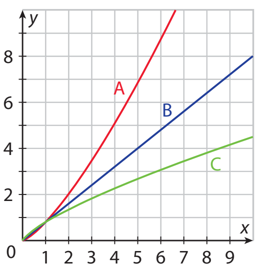
10. $\left(\sqrt[4]{2}\right)^7$
11. $\left(\sqrt{5x}\right)^3$
12. $\sqrt[5]{51^4}$
13. $\left(\sqrt{169}\right)^3$
14. $\left(\sqrt[4]{2v}\right)^3$
15. $\left(\sqrt[5]{n^2}\right)^2$
16. $\frac{1}{\left(\sqrt{3m}\right)^3}$
17. $\sqrt[7]{36^{14}}$
18. $\frac{1}{\left(\sqrt[4]{5p}\right)^7}$

Solve.

19. In every atom, electrons orbit the nucleus with a certain characteristic velocity known as the Fermi-Thomas velocity, equal to $\frac{Z^{\frac{2}{3}}}{137}c$, where Z is the number of protons in the nucleus and c is the speed of light. In terms of c, what is the characteristic Fermi-Thomas velocity of the electrons in Uranium, for which $Z = 92$?

23. Critical Thinking The graphs of three functions of the form $y = ax^{\frac{m}{n}}$ are shown for a specific value of a, where m and n are natural numbers. What can you conclude about the relationship of m and n for each graph? Explain.

See below.



24. Critical Thinking For a negative real number a, under what condition(s) on m and n $(n \neq 0)$ is $a^{\frac{m}{n}}$ a real number? Explain. (Assume $\frac{m}{n}$ is written in simplest form.)

See below.

Lesson Performance Task

The formula $W = 35.74 + 0.6215T - 35.75V^{\frac{4}{25}} + 0.4275TV^{\frac{4}{25}}$ relates the wind chill temperature W to the air temperature T in degrees Fahrenheit and the wind speed V in miles per hour. Find the wind chill to the nearest degree when the air temperature is 40 °F and the wind speed is 35 miles per hour. If the wind chill is about 23 °F to the nearest degree when the air temperature is 40 °F, what is the wind speed to the nearest mile per hour?

When the air temperature is 40 °F and the wind speed is 35 miles per hour, the wind chill is about 28 °F.

A wind speed of 80 miles per hour makes an air temperature of 40 °F feel like 23 °F.

23. For graph B, $m = n$, that is, $y = ax$. This is because the graph is that of a line, for which the exponent on x is 1 and the graph has a constant rate of change (slope). For graph A, $m > n$. This is because for a power greater than 1, the average rate of change of the graph increases as x increases, that is, the graph gets steeper. For graph C, $m < n$. This is because for a power less than 1, the average rate of change of the graph decreases as x increases, that is, the graph gets less steep.

24. If n is odd you can find a real number odd root for every real number, positive or negative. But if n is even (so m is odd since the fraction is in lowest terms), then you are trying to find an even root of a negative number (in $\sqrt[n]{a^m}$, a^m is negative), which is not possible.

Extension Activity

Have students derive an equation for the wind chill when the temperature is in degrees Celsius and the wind speed is in kilometers per hour. Have students explain how the two equations are different. Ask students how the unit conversion affects the exponents to which the variables are raised. The exponents remain the same. The unit conversion affects only the coefficients of the terms.

Lesson Performance Task Scoring Rubric

Points	Criteria
2	Student correctly solves the problem and explains his/her reasoning.
1	Student shows good understanding of the problem but does not fully solve or explain his/her reasoning.
0	Student does not demonstrate understanding of the problem.