10.3 Graphing Cube Root Functions

Essential Question: How can you use transformations of the parent cube root function to graph functions of the form $f(x) = a\sqrt[3]{(x-h)} + k$ or $g(x) = \sqrt[3]{\frac{1}{b}(x-h)} + k$?

Resource Locker

Explore Graphing and Analyzing the Parent Cube Root Function

The cube root parent function is $f(x) = \sqrt[3]{x}$. To graph $f(x)$, choose values of x and find corresponding values of y. Choose both negative and positive values of x.

Graph the function $f(x) = \sqrt[3]{x}$. Identify the domain and range of the function.

(A) Make a table of values.

x	y	x, y
−8	**−2** ?	**(−8, −2)** ?
−1	**−1** ?	**(−1, −1)** ?
0	**0** ?	**(0, 0)** ?
1	**1** ?	**(1, 1)** ?
8	**2** ?	**(8, 2)** ?

(B) Use the table to graph the function. **See below.**

(C) Identify the domain and range of the function.

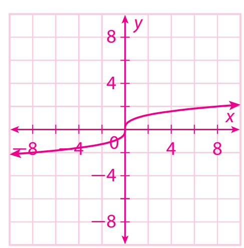
The domain is ? . **all real numbers**

The range is ? . **all real numbers**

(D) Does the graph of $f(x) = \sqrt[3]{x}$ have any symmetry?

The graph has ? . **point symmetry**

B.



Reflect

1. Can the radicand in a cube root function be negative?
Yes, because the cube of a negative number is negative.

Graphing Cube Root Functions

Learning Objective

Students will graph, analyze, and model with transformations of the cube root function.

Math Processes and Practices

MPP4 Mathematical Modeling

Language Objective

Describe how the graph of a cube root function differs from the graph of a square root function.

Online Resources

An extra example for each Explain section is available online.

Engage

Essential Question: How can you use transformations of the parent cube root function to graph functions of the form $f(x) = a\sqrt[3]{x-h} + k$ or $g(x) = \sqrt[3]{\frac{1}{b}(x-h)} + k$?

Possible answer: For , $f(x)$, the graph is a vertical stretch or compression of the parent graph by a factor of a. When $|a| > 1$, the graph is a vertical stretch; when $0 < |a| < 1$, the graph is a vertical compression. For $g(x)$ the graph is a horizontal stretch or compression of the parent graph by a factor of b. When $|b| > 1$, the graph is a horizontal stretch, and when $0 < |b| < 1$, the graph is a horizontal compression. h translates the graph horizontally, and k, vertically.

Preview: Lesson Performance Task

View the Engage section online. Discuss the photo and how an object's density can affect its size. Have students list variables that might be useful when calculating an object's size. Then preview the Lesson Performance Task.

Professional Development

Learning Progressions

Previously, students learned how parameters affect the graphs of square root functions. In this lesson, students analyze how the parameters a, b, h, and k, in functions of the form $f(x) = a\sqrt[3]{x-h} + k$ and $f(x) = \sqrt[3]{\frac{1}{b}(x-h)} + k$, affect the graph of the parent cube root function, $f(x) = \sqrt[3]{x}$. They then use this knowledge to graph cube root functions, and to analyze graphs to determine the functions they represent. This will help students when they work with radical functions with indices greater than 3. They will be able to distinguish between the attributes of functions with even indices and those with odd indices, and apply transformations to the graphs of those functions.

Explore

Graphing and Analyzing the Parent Cube Root Function

Integrate Technology

Students have the option of completing the Explore activity either in the book or online.

Questioning Strategies

How do you know that the graph of $f(x) = \sqrt[3]{x}$ does not have any horizontal asymptotes? As the value of x increases, the cube root of x increases without bound, and as the value of x decreases, the cube root of x decreases without bound, so there are no horizontal asymptotes. (As $x \to \infty$, $f(x) \to \infty$, and as $x \to -\infty$, $f(x) \to -\infty$.) The function is an increasing function, and the range is all real numbers.

Explain 1

Graphing Cube Root Functions

Integrate Math Processes and Practices Focus on Abstract and Quantitative Reasoning

MPP2 When graphing a cube-root function, students can create a table of values, and use the points from the table to help them to draw a more accurate curve. Discuss with students how to choose values of x that will make this work easier. Lead them to recognize that they need to choose values of x that make the *radicand* a perfect cube (positive or negative), not necessarily values of x that are themselves perfect cubes.

Explain 1 Graphing Cube Root Functions

Transformations of the Cube Root Parent Function $f(x) = \sqrt[3]{x}$

Transformation	$f(x)$ Notation	Examples
Vertical translation	$f(x) + k$	$y = \sqrt[3]{x} + 3$ 3 units up $y = \sqrt[3]{x} - 4$ 4 units down
Horizontal translation	$f(x - h)$	$y = \sqrt[3]{x - 2}$ 2 units right $y = \sqrt[3]{x + 1}$ 1 units left
Vertical stretch/compression	$af(x)$	$y = 6\sqrt[3]{x}$ vertical stretch by a factor of 6 $y = \frac{1}{2}\sqrt[3]{x}$ vertical compression by a factor of $\frac{1}{2}$
Horizontal stretch/ compression	$f\left(\frac{1}{b}x\right)$	$y = \sqrt[3]{\frac{1}{5}x}$ horizontal stretch by a factor of 5 $y = \sqrt[3]{3x}$ horizontal compression by a factor of $\frac{1}{3}$
Reflection	$-f(x)$ $f(-x)$	$y = -\sqrt[3]{x}$ across x-axis $y = \sqrt[3]{-x}$ across y-axis

For the function $f(x) = a\sqrt[3]{x - h} + k$, (h, k) is the graph's point of symmetry. Use the values of a, h, and k to draw each graph. Note that the point $(1, 1)$ on the graph of the parent function becomes the point $(1 + h, a + k)$ on the graph of the given function.

For the function $f(x) = \sqrt[3]{\frac{1}{b}(x - h)} + k$, (h, k) remains the graph's point of symmetry. Note that the point $(1, 1)$ on the graph of the parent function becomes the point $(b + h, 1 + k)$ on the graph of the given function.

Example 1 **Graph the cube root functions.**

Ⓐ Graph $g(x) = 2\sqrt[3]{x - 3} + 5$.

The transformations of the graph of $f(x) = \sqrt[3]{x}$ that produce the graph of $g(x)$ are:

- a vertical stretch by a factor of 2
- a translation of 3 units to the right and 5 units up

Choose points on $f(x) = \sqrt[3]{x}$ and find the transformed corresponding points on $g(x) = 2\sqrt[3]{x - 3} + 5$.

Graph $g(x) = 2\sqrt[3]{x - 3} + 5$ using the transformed points.

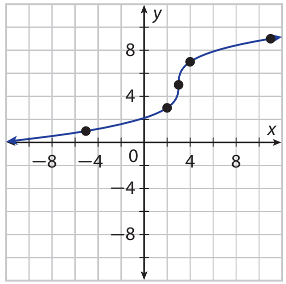
(See the table and graph on the next page.)

Collaborative Learning

Whole Class Activity

Project a large coordinate grid on the board. Have a student draw the graph of $f(x) = \sqrt[3]{x}$ on the grid. Select five students, and place each student's first initial on one of the following points on the graph: $(-8, -2)$, $(-1, -1)$, $(0, 0)$, $(1, 1)$, $(8, 2)$. Have the students point to "their" points on the projected graph. Then present a cube root function, such as $f(x) = -3\sqrt[3]{x + 1} + 4$ or $f(x) = \sqrt[3]{\frac{1}{2}(x - 3)} - 2$, and have each of the five students move his or her finger (or a pointer) to its new location on the graph of the given function. Have the other students in the class check for correctness. Then repeat the activity for other students using different functions.

$f(x) = \sqrt[3]{x}$	$g(x) = 2\sqrt[3]{x-3} + 5$
$(-8, -2)$	$(-5, 1)$
$(-1, -1)$	$(2, 3)$
$(0, 0)$	$(3, 5)$
$(1, 1)$	$(4, 7)$
$(8, 2)$	$(11, 9)$



(B) Graph $g(x) = \sqrt[3]{\frac{1}{2}(x-10)} + 4$.

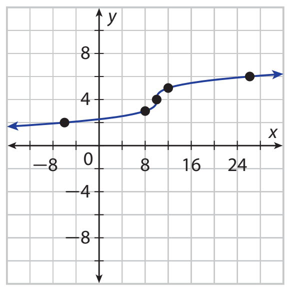
The transformations of the graph of $f(x) = \sqrt[3]{x}$ that produce the graph of $g(x)$ are:

- a horizontal stretch by a factor of 2
- a translation of 10 units to the right and 4 units up

Choose points on $f(x) = \sqrt[3]{x}$ and find the transformed corresponding points on $g(x) = \sqrt[3]{\frac{1}{2}(x-10)} + 4$.

Graph $g(x) = \sqrt[3]{\frac{1}{2}(x-10)} + 4$ using the transformed points.

$f(x) = \sqrt[3]{x}$	$g(x) = \sqrt[3]{\frac{1}{2}(x-10)} + 4$
$(-8, -2)$	$(-6, 2)$
$(-1, -1)$	$(8, 3)$
$(0, 0)$	$(10, 4)$
$(1, 1)$	$(12, 5)$
$(8, 2)$	$(26, 6)$

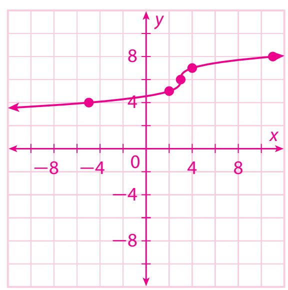


Your Turn

Graph the cube root function.

2. Graph $g(x) = \sqrt[3]{x-3} + 6$.

$f(x) = \sqrt[3]{x}$	$g(x) = \sqrt[3]{x-3} + 6$	
$(-8, -2)$	?	**$(-5, 4)$**
$(-1, -1)$	?	**$(2, 5)$**
$(0, 0)$	?	**$(3, 6)$**
$(1, 1)$	?	**$(4, 7)$**
$(8, 2)$	?	**$(11, 8)$**



Avoid Common Errors

Students may sometimes restrict the domain of cube root functions to values of the variable that make the radicand non-negative. Remind students that radicals with odd indices have real roots even when the radicand is negative. Therefore, they should graph cube root functions over the set of all real numbers.

Questioning Strategies

What effect do *h* and *k* have on the domain and range of a cube root function? None; the domain and range are both the set of all real numbers, regardless of the values of *h* and *k*.

Differentiate Instruction

Multiple Representations

Have students complete the table to examine the effects produced by *a* and *b*. Students can graph the functions to see the effects.

x	$f(x) = \sqrt[3]{x}$	$g(x) = 8\sqrt[3]{x}$	$g(x) = \sqrt[3]{8x}$	$g(x) = \sqrt[3]{\frac{1}{8}x}$
−64	−4	−32	−8	−2
−8	−2	−16	−4	−1
−1	−1	−8	−2	−0.5
0	0	0	0	0
1	1	8	2	0.5
8	2	16	4	1
64	4	32	8	2

Explain 2

Writing Cube Root Functions

Questioning Strategies

How can you identify the point to which (0, 0) from the graph of the parent function was translated in the graph of a cube root function? The point (0, 0) is translated to the point at which the curvature of the graph changes.

Avoid Common Errors

It is easy for students to forget that when indicating a horizontal stretch or compression of the parent graph, the reciprocal of the stretch or compression factor, and not the factor itself, is placed under the radical sign. Present a side-by-side comparison of a stretch and a compression to remind students of the proper procedure.

Integrate Technology

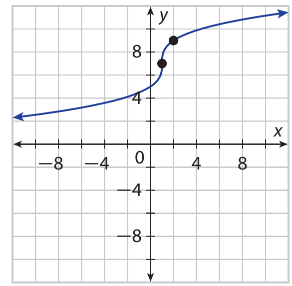
Students can use a graphing calculator to check their work. They can enter their functions, and use the graphing feature to check that the graphs of their functions match the given graph.

Explain 2 Writing Cube Root Functions

Given the graph of the transformed function $g(x) = a\sqrt[3]{\frac{1}{b}(x-h)} + k$, you can determine the values of the parameters by using the reference points $(-1, 1)$, $(0, 0)$, and $(1, 1)$ that you used to graph $g(x)$ in the previous example.

Example 2 **For the given graphs, write a cube root function.**

Ⓐ Write the function in the form $g(x) = a\sqrt[3]{x-h} + k$.



Identify the values of a, h, and k.

Identify the values of h and k from the point of symmetry.

$(h, k) = (1, 7)$, so $h = 1$ and $k = 7$.

Identify the value of a from either of the other two reference points $(-1, 1)$ or $(1, 1)$.

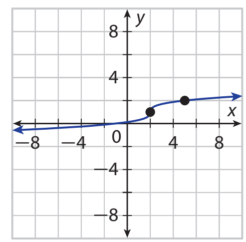
The reference point $(1, 1)$ has general coordinates $(h+1, a+k)$. Substituting 1 for h and 7 for k and setting the general coordinates equal to the actual coordinates gives this result:

$(h+1, a+k) = (2, a+7) = (2, 9)$, so $a = 2$.

$a = 2$ $\qquad$ $h = 1$ $\qquad$ $k = 7$

The function is $g(x) = 2\sqrt[3]{x-1} + 7$.

Ⓑ Write the function in the form $g(x) = \sqrt[3]{\frac{1}{b}(x-h)} + k$.



Identify the values of b, h, and k.

Identify the values of h and k from the point of symmetry.

$(h, k) = \left(2, \boxed{1}\right)$ so $h = 2$ and $k = \boxed{1}$.

Identify the value of b from either of the other two reference points.

The rightmost reference point has general coordinates $(b+h, 1+k)$. Substituting 2 for h and 1 for k and setting the general coordinates equal to the actual coordinates gives this result:

$\left(b+h, 1+\boxed{1}\right) = \left(b+2, \boxed{2}\right) = (5, 2)$, so $b = \boxed{3}$.

$b = \boxed{3}$ $\qquad$ $h = \boxed{2}$ $\qquad$ $k = \boxed{1}$

The function is $g(x) = \sqrt[3]{\frac{1}{3}(x-2)} + 1$.

Language Support EL

Communicate Math

Have students work in pairs to complete a chart similar to the following, comparing and contrasting square root and cube root functions.

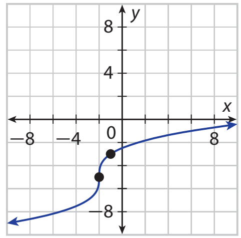
Type of Function	Example	Description	Similarities	Differences

Your Turn

For the given graphs, write a cube root function.

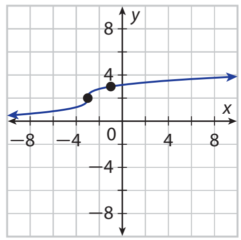
3. Write the function in the form $g(x) = a\sqrt[3]{x - h} + k$.

$g(x) = 2\sqrt[3]{(x + 2)} - 5$



4. Write the function in the form $g(x) = \sqrt[3]{\frac{1}{b}(x - h)} + k$.

$g(x) = \sqrt[3]{\frac{1}{2}(x + 3)} + 2$



Explain 3 Modeling with Cube Root Functions

You can use cube root functions to model real-world situations.

Example 3

Ⓐ The shoulder height h (in centimeters) of a particular elephant is modeled by the function $h(t) = 62.1\sqrt[3]{t} + 76$, where t is the age (in years) of the elephant. Graph the function and examine its average rate of change over the equal t-intervals $(0, 20)$, $(20, 40)$, and $(40, 60)$. What is happening to the average rate of change as the t-values of the intervals increase? Use the graph to find the height when $t = 35$.

Graph $h(t) = 62.1\sqrt[3]{t} + 76$.

The graph is the graph of $f(x) = \sqrt[3]{x}$ translated up 76 and stretched vertically by a factor of 62.1. Graph the transformed points $(0, 76)$, $(8, 200.2)$, $(27, 262.3)$, and $(64, 324.4)$. Connect the points with a smooth curve.

First interval:

$$\text{Average Rate of change} \approx \frac{244.6 - 76}{20 - 0}$$
$$= 8.43$$

Explain 3

Modeling with Cube Root Functions

Questioning Strategies

What information about the situation is better illustrated by the graph than by the function rule? The graph makes it easier to see the behavior of the function as the values of x increase.

Integrate Math Processes and Practices
Focus on Problem Solving

MPP1 Instruct students to find the inverse of the function in the example. Have them compare the two functions. Then have them discuss the types of questions that could be answered better by the inverse function than by the original function.

Elaborate Answers

6. Answers may vary. Sample answer: You can take the cube root of negative numbers, zero, and positive numbers, so the value of the radicand in $\sqrt[3]{x}$ can be positive, negative, or zero, which makes the domain all real numbers.

8. The graph of $f(x)$ is a vertical stretch or compression of the graph of the parent function by a factor of a. When $|a| > 1$, the graph of $f(x)$ is a vertical stretch, and when $0 < |a| < 1$, the graph of $f(x)$ is a vertical compression. The graph of $g(x)$ is a horizontal stretch or compression of the graph of the parent function by a factor of b. When $|b| > 1$ the graph of $g(x)$ is a horizontal stretch, and when $0 < |b| < 1$ the graph of $g(x)$ is a horizontal compression. In both cases, the graphs are translated left or right by h and up or down by k.

Second interval:

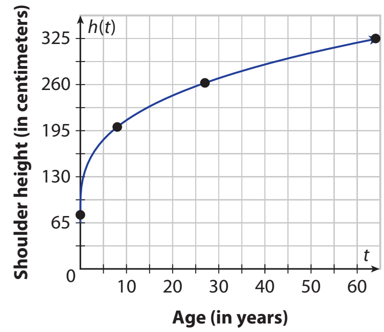
$$\text{Average Rate of change} \approx \frac{288.4 - 244.6}{40 - 20}$$
$$= 2.19$$

Third interval:

$$\text{Average Rate of change} \approx \frac{319.1 - 288.4}{60 - 40}$$
$$= 1.54$$

The average rate of change is becoming less.

Drawing a vertical line up from 35 gives a value of about 280 cm.

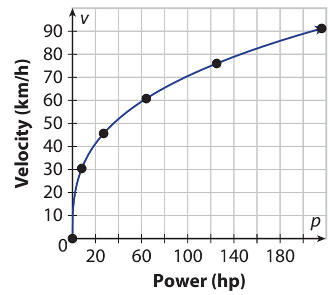


B The velocity of a 1400-kilogram car at the end of a 400-meter run is modeled by the function $v = 15.2\sqrt[3]{p}$, where v is the velocity in kilometers per hour and p is the power of its engine in horsepower. Graph the function and examine its average rate of change over the equal p-intervals $(0, 60)$, $(60, 120)$, and $(120, 180)$. What is happening to the average rate of change as the p-values of the intervals increase? Use the function to find the velocity when p is 100 horsepower.

Graph $v = 15.2\sqrt[3]{p}$.

The graph is the graph of $f(x) = \sqrt[3]{x}$ stretched vertically by a factor of 15.2. Graph the transformed points $(0, 0)$, $(8, 30.4)$, $(27, 45.6)$, $(64, 60.8)$, $(125, 76)$, and $(216, 91.2)$.

Connect the points with a smooth curve.



The average rate of change over the interval $(0, 60)$

is $\dfrac{\boxed{59.5} - \boxed{0}}{60 - 0}$ which is about 0.99.

The average rate of change over the interval $(60, 120)$

is $\dfrac{\boxed{75.0} - \boxed{59.5}}{120 - 60}$ which is about 0.26.

The average rate of change over the interval $(120, 180)$

is $\dfrac{\boxed{85.8} - \boxed{75.0}}{180 - 120}$ which is about 0.18.

The average rate of change is becoming less.

Evaluate Answers

3. The graph of $g(x) = \sqrt[3]{x} + 6$ is the graph of $f(x) = \sqrt[3]{x}$ translated 6 units up.

4. The graph of $g(x) = \sqrt[3]{x - 5}$ is the graph of $f(x) = \sqrt[3]{x}$ translated 5 units right.

5. The graph of $g(x) = \frac{1}{3}\sqrt[3]{-x}$ is the graph of $f(x) = \sqrt[3]{x}$ compressed vertically by a factor of $\frac{1}{3}$ and reflected across the y-axis.

6. The graph of $g(x) = \sqrt[3]{5x}$ is the graph of $f(x) = \sqrt[3]{x}$ compressed horizontally by a factor of $\frac{1}{5}$.

7. The graph of $g(x) = -2\sqrt[3]{x} + 3$ is the graph of $f(x) = \sqrt[3]{x}$ reflected across the x-axis, vertically stretched by a factor of 2, and then translated 3 units up.

8. The graph of $g(x) = \sqrt[3]{x + 4} - 3$ is the graph of $f(x) = \sqrt[3]{x}$ translated 4 units left and 3 units down.

Substitute $p = 100$ in the function.

$v = 15.2\sqrt[3]{p}$

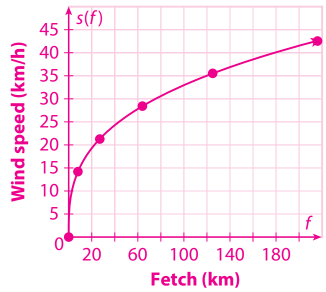
$v = 15.2\sqrt[3]{\boxed{100}}$

$v \approx 15.2\left(\boxed{4.64}\right)$

$v \approx \boxed{70.5}$

The velocity is about 70.5 km/h.

5.



Your Turn

5. The fetch is the length of water over which the wind is blowing in a certain direction. The function $s(f) = 7.1\sqrt[3]{f}$, relates the speed of the wind s in kilometers per hour to the fetch f in kilometers. Graph the function and examine its average rate of change over the intervals $(20, 80)$, $(80, 140)$, and $(140, 200)$. What is happening to the average rate of change as the f-values of the intervals increase? Use the function to find the speed of the wind when $f = 64$.
first interval: $\approx$ 0.19; second interval: $\approx$ 0.11; third interval: $\approx$ 0.08
The average rate of change is becoming less. The speed of the wind is about 28.4 km/h.

Elaborate

6. Discussion Why is the domain of $f(x) = \sqrt[3]{x}$ all real numbers? **See margin.**

7. Identify which transformations (stretches or compressions, reflections, and translations) of $f(x) = \sqrt[3]{x}$ change the following attributes of the function.

a. Location of the point of symmetry **a. Vertical translations $(k \neq 0)$ and horizontal translations $(h \neq 0)$ change the location of the point of symmetry.**

b. Symmetry about a point **No transformations change the function's symmetry about a point.**

8. Essential Question Check-In How do parameters a, b, h, and k effect the graphs of $f(x) = a\sqrt[3]{(x - h)} + k$ and $g(x) = \sqrt[3]{\frac{1}{b}(x - h)} + k$? **See margin.**

Evaluate: Homework and Practice

Personal Math Trainer
- Online Homework
- Hints and Help
- Extra Practice

1. The domain is all real numbers. The range is all real numbers.

1. Graph the function $g(x) = \sqrt[3]{x} + 3$. Identify the domain and range of the function.

2. The domain is all real numbers. The range is all real numbers.

2. Graph the function $g(x) = \sqrt[3]{x} - 5$. Identify the domain and range of the function.

1–2. For graphs, see Additional Answers.

Describe how the graph of the function compares to the graph of $f(x) = \sqrt[3]{x}$.

3–8. See margin.

3. $g(x) = \sqrt[3]{x} + 6$

4. $g(x) = \sqrt[3]{x - 5}$

5. $g(x) = \frac{1}{3}\sqrt[3]{-x}$

6. $g(x) = \sqrt[3]{5x}$

7. $g(x) = -2\sqrt[3]{x} + 3$

8. $g(x) = \sqrt[3]{x + 4} - 3$

Elaborate

Summarize The Lesson

How is the graph of $f(x) = a\sqrt[3]{x - h} + k$ related to the graph of $f(x) = \sqrt[3]{x}$? The point at which the curvature of the graph changes is at (h, k) instead of $(0, 0)$. It is stretched vertically by a factor of a if $|a| > 1$, or compressed vertically by a factor of a if $0 < |a| < 1$. If $a < 0$, then the graph is decreasing instead of increasing.

Evaluate

Personal Math Trainer

Assignment Guide

Level	Concepts and Skills	Practice
Basic	Explore	Exercises 1–2
	Example 1	Exercises 3–11
	Example 2	Exercise 12
	Example 3	Exercise 15
	H.O.T.	Exercise 22
Average	Explore	Exercises 1–2
	Example 1	Exercises 3–11
	Example 2	Exercises 12–13
	Example 3	Exercise 15
	H.O.T.	Exercises 22–23
Advanced	Explore	Exercise 2
	Example 1	Exercises 3–11
	Example 2	Exercises 13–14
	Example 3	Exercise 16
	H.O.T.	Exercises 22–24

Real World Problems

Exercise	Depth of Knowledge (D.O.K.)	Math Processes and Practices
1–2	**1** Recall of Information	**MPP4** Mathematical Modeling
3–8	**2** Skills/Concepts	**MPP2** Abstract and Quantitative Reasoning
9–14	**2** Skills/Concepts	**MPP6** Using Precise Mathematical Language
15–16	**2** Skills/Concepts	**MPP4** Mathematical Modeling
17	**2** Skills/Concepts	**MPP6** Using Precise Mathematical Language
18	**2** Skills/Concepts	**MPP4** Mathematical Modeling

Integrate Math Processes and Practices
Focus on Using Precise Mathematical Language

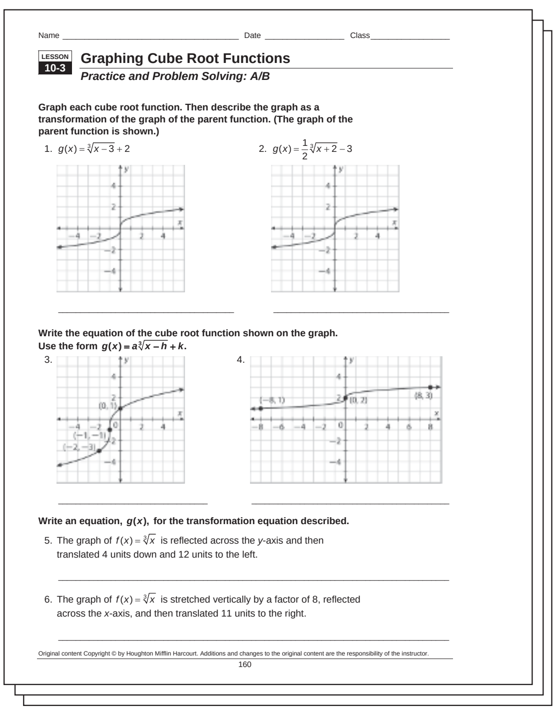
MPP6 Students can check that they've used accurate language to describe the transformations by graphing the given function and checking to see if the graph matches the description. If students are having difficulty, you may want to suggest that they work on these problems with a partner, verbalizing and then refining their descriptions with the partner.

Multiple Representations

Encourage students to make a table of values for the function in order to find additional points on the graph. This will help them to draw a more accurate curve.

Online Resources

- Practice and Problem Solving (three forms)
- Reteach
- Reading Strategies
- Success for English Learners


Name ______ Date ______ Class ______

LESSON 10-3 **Graphing Cube Root Functions**
Practice and Problem Solving: A/B

Graph each cube root function. Then describe the graph as a transformation of the graph of the parent function. (The graph of the parent function is shown.)

1. $g(x) = \sqrt[3]{x-3} + 2$
2. $g(x) = \frac{1}{2}\sqrt[3]{x+2} - 3$

Write the equation of the cube root function shown on the graph. Use the form $g(x) = a\sqrt[3]{x-h} + k$.

3.
4.

Write an equation, $g(x)$, for the transformation equation described.

5. The graph of $f(x) = \sqrt[3]{x}$ is reflected across the y-axis and then translated 4 units down and 12 units to the left.

6. The graph of $f(x) = \sqrt[3]{x}$ is stretched vertically by a factor of 8, reflected across the x-axis, and then translated 11 units to the right.

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Graph the cube root functions.

9. $g(x) = 3\sqrt[3]{x+4}$ **See below.**

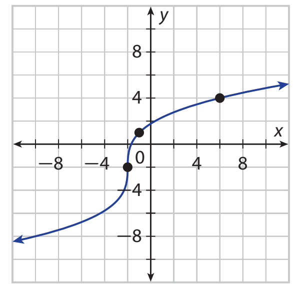
10. $g(x) = 2\sqrt[3]{x} + 3$ **See below.**

11. $g(x) = \sqrt[3]{x-3} + 2$ **See margin.**

For the given graphs, write a cube root function.

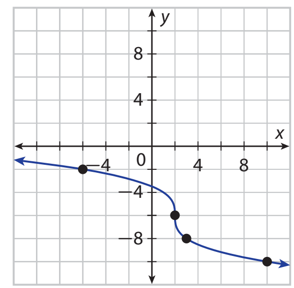
12. Write the function in the form $g(x) = a\sqrt[3]{x-h} + k$.

$g(x) = 3\sqrt[3]{x+2} - 2$



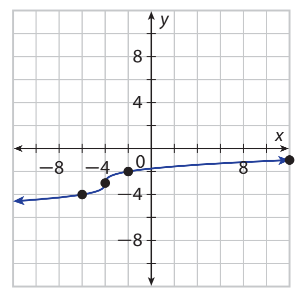
13. Write the function in the form $g(x) = a\sqrt[3]{x-h} + k$.

$g(x) = -2\sqrt[3]{x-2} - 6$



14. Write the function in the form $g(x) = \sqrt[3]{\frac{1}{b}(x-h)} + k$.

$g(x) = \sqrt[3]{\frac{1}{2}(x+4)} - 3$



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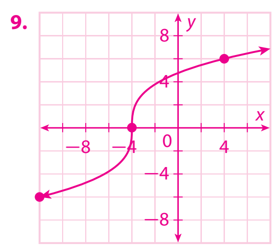

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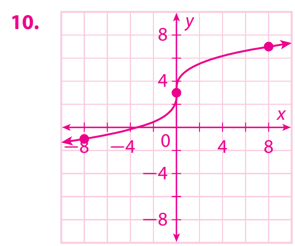


Exercise	Depth of Knowledge (D.O.K.)	Math Processes and Practices
19–21	**2** Skills/Concepts	**MPP2** Abstract and Quantitative Reasoning
22	**3** Strategic Thinking H.O.T.	**MPP2** Abstract and Quantitative Reasoning
23–24	**3** Strategic Thinking H.O.T.	**MPP3** Using and Evaluating Logical Reasoning

15. The length of the side of a cube is modeled by $s = \sqrt[3]{V}$. Graph the function. Use the graph to find s when $V = 48$.

See Additional Answers for graph.

When $V = 48$, $s \approx 3.6$.

17. 1. Graph the parent function $f(x) = \sqrt[3]{x}$.

2. Translate the graph of $f(x) = \sqrt[3]{x}$ 8 units to the left.

3. Translate the resulting graph 11 units down.

16. The radius of a stainless steel ball, in centimeters, can be modeled by $r(m) = 0.31\sqrt[3]{m}$, where m is the mass of the ball in grams. Use the function to find r when $m = 125$.

A stainless steel ball bearing with a mass of 125 grams has a radius of 1.55 centimeters.

17. Describe the steps for graphing $g(x) = \sqrt[3]{x+8} - 11$. **See above.**

18. **Modeling** Write a situation that can be modeled by a cube root function. Give the function. **See below.**

19. Find the y-intercept for the function $y = a\sqrt[3]{x-h} + k$. $\boldsymbol{y = a\sqrt[3]{-h} + k}$ **or** $\boldsymbol{y = -ah^{1/3} + k}$

20. Find the x-intercept for the function $y = a\sqrt[3]{x-h} + k$.

21. Describe the translation(s) used to get $g(x) = \sqrt[3]{x-9} + 12$ from $f(x) = \sqrt[3]{x}$. Select all that apply. **A and G**

20. $\boldsymbol{x = \frac{(-k)^3}{a^3} + h}$

A. translated 9 units right

B. translated 9 units left

C. translated 9 units up

D. translated 9 units down

E. translated 12 units right

F. translated 12 units left

G. translated 12 units up

H. translated 12 units down

H.O.T. Focus on Higher Order Thinking **22–23. See Additional Answers.**

22. **Explain the Error** Tim says that to graph $g(x) = \sqrt[3]{x-6} + 3$, you need to translate the graph of $f(x) = \sqrt[3]{x}$ 6 units to the left and then 3 units up. What mistake did he make?

23. **Communicate Mathematical Ideas** Why does the square root function have a restricted domain but the cube root function does not?

24. **Justify Reasoning** Does a horizontal translation and a vertical translation of the function $f(x) = \sqrt[3]{x}$ affect the function's domain or range? Explain.

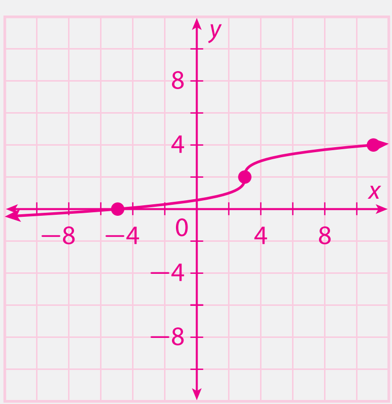
No, the domain and the range of $f(x) = \sqrt[3]{x}$ is all real numbers. After a horizontal or vertical translation, the domain or range will still be all real numbers.

18. Answers may vary. Sample answer: The radius of a sphere as a function of its volume.

$$r = \sqrt[3]{\frac{3V}{4\pi}}$$

Answers

11.



Cooperative Learning

Have students work in small groups. Have each group make a poster showing how to graph a function of the form $f(x) = a\sqrt[3]{x-h} + k$ and a function of the form $f(x) = \sqrt[3]{\frac{1}{b}(x-h)} + k$. Give each group different functions to graph. Then have each group present its poster to the rest of the class, explaining each step.

Visual Cues

Suggest that students draw the graph of the parent cube root function on the same coordinate grid as the given graph so that they can see whether the given graph represents a stretch or compression of the graph of the parent function.

Avoid Common Errors

When graphing a function of the form $f(x) = a\sqrt[3]{x-h} + k$ with $a < 0$, some students may perform the translation first and then perform the reflection. Help them to see that the function indicates a vertical translation of the graph of $f(x) = a\sqrt[3]{x-h}$ by k units, so the graph needs to be reflected before it is translated vertically.

Peer-to-Peer Discussion

Ask students to discuss with a partner how they can use the rule for a cube root function to determine the x- and y-intercepts of its graph. To find the x-intercept, set the rule equal to 0 and solve for x. To find the y-intercept, substitute 0 for x and solve for y.

Connect Vocabulary EL

Ask students to articulate what a square root function is and how it is related to a quadratic function. Then have them explain what a cube root function is and how it is related to a cubic function.

Journal

Have students compare and contrast the process of graphing a cube root function with that of graphing a square root function.

Avoid Common Errors

Some students may not take the cube root of the constant factor when solving for r. The cube root must be applied to coefficients on both sides of the equation. When solving for r in $r^3 = \frac{1}{12\pi}m$, the cube root must be applied to both $\frac{1}{12\pi}$ and m: $r = \sqrt[3]{\frac{1}{12\pi}}\sqrt[3]{m}$.

Integrate Math Processes and Practices
Focus on Abstract and Quantitative Reasoning

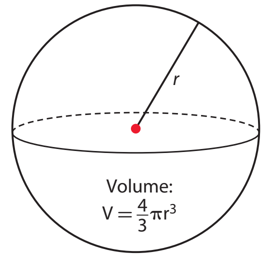
MPP2 Have students discuss whether the size of an object always varies as the cube root of its mass. Have students consider different shapes and determine the size of the object in terms of its mass. Ask students to explain when the size might be the cube root of its mass and when it might be a different function.

Lesson Performance Task Scoring Rubric

Points	Criteria
2	Student correctly solves the problem and explains his/her reasoning.
1	Student shows good understanding of the problem but does not fully solve or explain his/her reasoning.
0	Student does not demonstrate understanding of the problem.

Lesson Performance Task

The side length of a 243-gram copper cube is 3 centimeters. Use this information to write a model for the radius of a copper sphere as a function of its mass. Then, find the radius of a copper sphere with a mass of 50 grams. How would changing the material affect the function?



$r(m) \approx \sqrt[3]{\frac{1}{12\pi}m} \approx 0.3\sqrt[3]{m}$

The radius of a 50-gram copper sphere is about 1.1 centimeters.
Changing the material of the sphere changes the function, because when density changes, the function $m(r)$ changes.

Extension Activity

Have students write a general equation for the radius r with the density D as a variable. $r(m) = \sqrt[3]{\frac{3}{4\pi D}m}$ Then have them explain how the density D affects the graph of the function $r(m)$. If $0 < D < 1$, then D causes a horizontal compression of the graph. If $D > 1$, then D causes a horizontal stretch of the graph.