10.2 Graphing Square Root Functions

Essential Question: How can you use transformations of a parent square root function to graph functions of the form $g(x) = a\sqrt{(x-h)} + k$ or $g(x) = \sqrt{\frac{1}{b}(x-h)} + k$?

Resource Locker

Explore Graphing and Analyzing the Parent Square Root Function

Although you have seen how to use imaginary numbers to evaluate square roots of negative numbers, graphing complex numbers and complex valued functions is beyond the scope of this course. For purposes of graphing functions based on the square roots (and in most cases where a square root function is used in a real-world example), the domain and range should both be limited to real numbers.

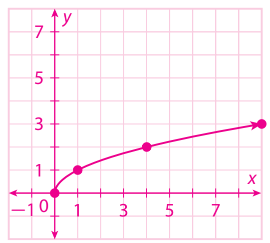
The square root function is the inverse of a quadratic function with a domain limited to positive real numbers. The quadratic function must be a one-to-one function in order to have an inverse, so the domain is limited to one side of the vertex. The square root function is also a one-to-one function as all inverse functions are.

(A) The domain of the square root function (limited to real numbers) is given by $\{x \mid x \geq$? $\}$. **0**

(B) Fill in the table.

x	$f(x) = \sqrt{x}$
0	? **0**
1	? **1**
4	? **2**
9	? **3**

(C) Plot the points on a coordinate grid, and connect them with a smooth curve.



(D) Recall that this function is the inverse of the parent quadratic $\left(f(x) = x^2\right)$ with a domain limited to the nonnegative real numbers. Write the range of this square root function:

$\{y \mid y \geq$? $\}$. **0**

(E) The graph appears to be getting flatter as x increases, indicating that the rate of change ? as x increases. **decreases**

(F) Describe the end behavior of the square root function, $f(x) = \sqrt{x}$. $f(x) \to$? as $x \to$?

∞; ∞

Reflect **1. See margin. 2. See Additional Answers.**

1. **Discussion** Why does the end behavior of the square root function only need to be described at one end?
2. The solution to the equation $x^2 = 4$ is sometimes written as $x = \pm 2$. Explain why the inverse of $f(x) = x^2$ cannot similarly be written as $g(x) = \pm\sqrt{x}$ in order to use all reals as the domain of $f(x)$.

Professional Development

Learning Progressions

Students have learned how parameters affect the graphs of quadratic functions, absolute value functions, and rational functions. In this lesson, these concepts are extended to the graphs of square root functions. Students analyze how the parameters a, b, h, and k, in functions of the form $f(x) = a\sqrt{x-h} + k$ and $f(x) = \sqrt{\frac{1}{b}(x-h)} + k$ affect the graph of the parent square root function, $f(x) = \sqrt{x}$. They then use this knowledge to graph square root functions, and to analyze graphs to determine the functions they represent.

Graphing Square Root Functions

Learning Objective

Students will graph, analyze, and model with transformations of the square root function.

Math Processes and Practices

MPP4 Mathematical Modeling

Language Objective

Discuss with a partner how the graphs of square root functions compare with quadratic functions.

Online Resources

An extra example for each Explain section is available online.

Engage

Essential Question: How can you use transformations of a parent square root function to graph functions of the form $f(x) = a\sqrt{x-h} + k$ or $g(x) = \sqrt{\frac{1}{b}(x-h)} + k$?

Possible answer: You can use the parameters a, b, h, and k to transform points on the parent function and use those transformed points to draw the graph of $f(x)$ or $g(x)$.

Preview: Lesson Performance Task

View the Engage section online. Discuss the photo and how a representative sample can give information about the entire batch. Then preview the Lesson Performance Task.

Answers

1. The function is not defined for negative values of x, and so its behavior as x approaches negative infinity is also not defined. The discrete point (0, 0) does not describe end behavior even though the function "ends" there.

Explore 1

Graphing and Analyzing the Parent Square Root Function

Integrate Technology

Students have the option of completing the Explore activity either in the book or online.

Questioning Strategies

How do you know that the graph of $f(x) = \sqrt{x}$ does not have a horizontal asymptote? As the value of x increases, the square root of x increases without bound, so there is no horizontal asymptote. (As $x \to \infty$, $f(x) \to \infty$.) The function is an increasing function, and the range is all non-negative real numbers.

Explore 2

Predicting the Effects of Parameters on the Graphs of Square Root Functions

Questioning Strategies

For functions of the form $f(x) = a\sqrt{x}$, why does a value of a that is between 0 and 1 create a vertical compression of the graph of the parent function? The function value for each x-value is multiplied by a number less than 1, making it less than the corresponding value for the parent function; this pulls the graph closer to the x-axis.

Integrate Math Processes and Practices

Focus on Problem Solving

MPP1 Discuss with students how they can use what they know about how the various parameters affect the graphs of quadratic functions to make predictions about how the parameters will affect the graphs of square root functions.

Explore 2 Predicting the Effects of Parameters on the Graphs of Square Root Functions

You have learned how to transform the graph of a function using reflections across the x- and y-axes, vertical and horizontal stretches and compressions, and translations. Here, you will apply those transformations to the graph of the square root function $f(x) = \sqrt{x}$.

When transforming the parent function $f(x) = \sqrt{x}$, you can get functions of the form

$g(x) = a\sqrt{(x-h)} + k$ or $g(x) = \sqrt{\frac{1}{b}(x-h)} + k$.

For each parameter, predict the effect on the graph of the parent function, and then confirm your prediction with a graphing calculator.

A Predict the effect of the parameter, h, on the graph of $g(x) = \sqrt{x-h}$ for each function.

a. $g(x) = \sqrt{x-2}$: The graph is a ? of the graph of $f(x)$? 2 units. **translation; right**

b. $g(x) = \sqrt{x+2}$: The graph is a ? of the graph of $f(x)$? 2 units. **translation; left**

Check your answers using a graphing calculator.

B Predict the effect of the parameter k on the graph of $g(x) = \sqrt{x} + k$ for each function.

a. $g(x) = \sqrt{x} + 2$: The graph is a ? of the graph of $f(x)$? 2 units. **translation; up**

b. $g(x) = \sqrt{x} - 2$: The graph is a ? of the graph of $f(x)$? 2 units. **translation; down**

Check your answers using a graphing calculator.

C Predict the effect of the parameter a on the graph of $g(x) = a\sqrt{x}$ for each function.

a. $g(x) = 2\sqrt{x}$: The graph is a ? stretch of the graph of $f(x)$ by a factor of ? . **vertical; 2**

b. $g(x) = \frac{1}{2}\sqrt{x}$: The graph is a ? compression of the graph of $f(x)$ by a factor of ? . **vertical; $\frac{1}{2}$**

c. $g(x) = -\frac{1}{2}\sqrt{x}$: The graph is a ? compression of the graph of $f(x)$ by a factor of ? as well as a ? across the ? . **vertical; $\frac{1}{2}$; reflection; *x*-axis**

d. $g(x) = -2\sqrt{x}$: The graph is a ? stretch of the graph of $f(x)$ by a factor of ? as well as a ? across the ? . **vertical; 2; reflection; *x*-axis**

Check your answers using a graphing calculator.

D Predict the effect of the parameter, b, on the graph of $g(x) = \sqrt{\frac{1}{b}x}$ for each function.

a. $g(x) = \sqrt{\frac{1}{2}x}$: The graph is a ? stretch of the graph of $f(x)$ by a factor of ? . **horizontal; 2**

b. $g(x) = \sqrt{2x}$: The graph is a ? compression of the graph of $f(x)$ by a factor of ? . **horizontal; $\frac{1}{2}$**

c. $g(x) = \sqrt{-\frac{1}{2}x}$: The graph is a ? stretch of the graph of $f(x)$ by a factor of ? as well as a ? across the ? . **horizontal; 2; reflection; *y*-axis**

d. $g(x) = \sqrt{-2x}$: The graph is a ? compression of the graph of $f(x)$ by a factor of ? as well as a ? across the ? . **horizontal; $\frac{1}{2}$; reflection; *y*-axis**

Check your answers using a graphing calculator.

Collaborative Learning

Peer-to-Peer Activity

Have students work in pairs. Have each student write a function of the form $f(x) = a\sqrt{x-h} + k$ but keep it hidden from the partner. Have students graph their functions on graph paper, exchange them with partners, and try to determine the function represented by the partner's graph. Have partners check each other's work. Then have the students repeat the activity using functions of the form $f(x) = \sqrt{\frac{1}{b}(x-h)} + k$.

Reflect

3. **Discussion** Describe what the effect of each of the transformation parameters is on the domain and range of the transformed function. **See margin.**

Explain 1 Graphing Square Root Functions

When graphing transformations of the square root function, it is useful to consider the effect of the transformation on two reference points, $(0, 0)$ and $(1, 1)$, that lie on the parent function, and where they map to on the transformed function, $g(x)$.

$f(x) = \sqrt{x}$		$g(x) = a\sqrt{x-h} + k$		$g(x) = \sqrt{\frac{1}{b}(x-h)} + k$	
x	y	x	y	x	y
0	0	h	k	h	k
1	1	$h+1$	$k+a$	$h+b$	$k+1$

The transformed reference points can be found by recognizing that the initial point of the graph is translated from $(0, 0)$ to (h, k). When $g(x)$ involves the parameters a, h, and k, the second transformed reference point is 1 unit to the right of (h, k) and $|a|$ units up or down from (h, k), depending on the sign of a. When $g(x)$ involves the parameters b, h, and k, the second transformed reference point is $|b|$ units left or right from (h, k), depending on the sign of b, and 1 unit above (h, k).

Transformations of the square root function also affect the domain and range. In order to work with real valued inputs and outputs, the domain of the square root function cannot include values of x that result in a negative-valued expression. Negative values of x can be in the domain, as long as they result in nonnegative values of the expression that is inside the square root. Similarly, the value of the square root function is positive by definition, but multiplying the square root function by a negative number, or adding a constant to it changes the range and can result in negative values of the transformed function.

Example 1 **For each of the transformed square root functions, find the transformed reference points and use them to plot the transformed function on the same graph with the parent function. Describe the domain and range using set notation.**

A $g(x) = 2\sqrt{x-3} - 2$

To find the domain:

Square root input must be nonnegative. $x - 3 \geq 0$

Solve the inequality for x. $x \geq 3$

The domain is $\{x \mid x \geq 3\}$.

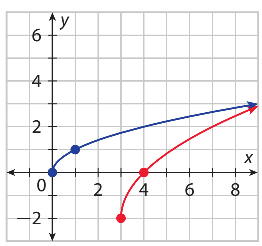
To find the range:

The square root function is nonnegative. $\sqrt{x-3} \geq 0$

Multiply by 2 $2\sqrt{x-3} \geq 0$

Subtract 2. $2\sqrt{x-3} - 2 \geq -2$

The expression on the left is $g(x)$. $g(x) \geq -2$



Differentiate Instruction

Kinesthetic Experience

Have students work in pairs. Have each pair place a transparency sheet over a sheet of graph paper with the axes labeled. Have them graph the radical function $f(x) = a\sqrt{x-h} + k$ on the transparency for $h = 0$ and $k = 0$. Then have them move the transparency to represent changes in h and k, and write the function that represents the transformation. Students can use a graphing calculator to check their answers.

Explain 1

Graphing Square Root Functions

Questioning Strategies

What is an example of a negative value of x that is in the domain of a square root function? Why is it part of the domain? In the function $g(x) = \sqrt{x+10}$, x can be −6 because it results in a nonnegative square root, $g(x) = \sqrt{-6+10} = \sqrt{4}$.

What effect do h and k have on the domain and range of $f(x) = a\sqrt{x-h} + k$? The domain is all real numbers greater than or equal to h. The range is all real numbers greater than or equal to k if $a > 0$, and less than or equal to k if $a < 0$.

Integrate Math Processes and Practices
Focus on Using Mathematical Tools

MPP5 Students can use a graphing calculator to help them better understand the effects of a stretch or a compression. Have them graph both the parent function and the given function on the calculator so they can see how the rate of change of the function is reflected in the graph.

Answers

3. Each value in the domain shifts left by $|h|$ if h is positive and right by $|h|$ if h is negative. h has no effect on the range. If a is negative, it changes the sign of each value in the range, but a has no effect on the domain. Each value in the range shifts up by $|k|$ if k is positive and down by $|k|$ if k is negative. k has no effect on the domain. If b is negative, it changes the sign of each value in the domain, but b has no effect on the range.

Peer-to-Peer Discussion

Ask students to discuss with a partner the difference between a vertical stretch and a horizontal stretch. Have them use graphs exhibiting each of these attributes to describe the comparison in their own words. Then ask them to do the same for vertical and horizontal compressions. Have them share their descriptions with the class.

Explain 2

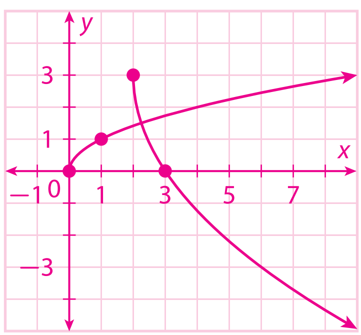
Writing Square Root Functions

Questioning Strategies

How can you tell the signs of h and k by looking at the graph? You can see which way the graph of the parent function has been translated. If it has been translated to the right, h is positive. If it has been translated to the left, h is negative. If it has been translated up, k is positive. If it has been translated down, k is negative.

Answers

4.



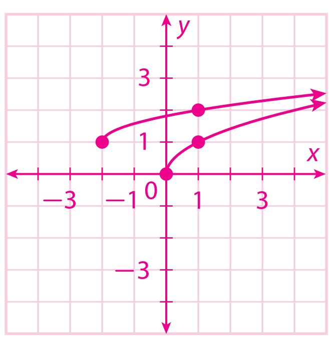
$(0, 0) \rightarrow (2, 3)$

$(1, 1) \rightarrow (3, 0)$

The domain is $\{x \mid x \geq 2\}$.

The range is $\{y \mid y \leq 3\}$.

5.



$(0, 0) \rightarrow (-2, 1)$

$(1, 1) \rightarrow (1, 2)$

The domain is $\{x \mid x \geq -2\}$.

The range is $\{y \mid y \geq 1\}$.

Since $g(x)$ is greater than or equal to -2 for all x in the domain, the range is $\{y \mid y \geq -2\}$.

$(0, 0) \rightarrow (3, -2)$

$(1, 1) \rightarrow (4, 0)$

B $g(x) = \sqrt{-\frac{1}{2}(x-2)} + 1$

To find the domain:

Square root input must be nonnegative. $-\frac{1}{2}(x-2) \geq \boxed{0}$

Multiply both sides by -2. $x - 2 \boxed{\leq} 0$

Add 2 to both sides. $\boxed{x} \leq 2$

Expressed in set notation, the domain is $\{x \mid \boxed{x \leq 2}\}$.

To find the range:

The square root function is nonnegative. $\sqrt{-\frac{1}{2}(x-2)} \boxed{\geq} 0$

Add 1 to both sides $\sqrt{-\frac{1}{2}(x-2)} + 1 \geq \boxed{1}$

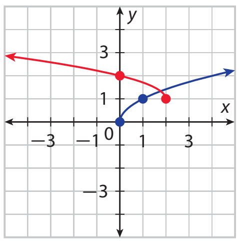
Substitute in $\boxed{g(x)}$. $g(x) \geq 1$

Since $g(x)$ is greater than 1 for all x in the domain,

the range (in set notation) is $\{y \mid \boxed{y \geq 1}\}$.

$(0, 0) \rightarrow \boxed{(2, 1)}$

$(1, 1) \rightarrow \boxed{(0, 2)}$



Your Turn

For each of the transformed square root functions, find the transformed reference points and use them to plot the transformed function on the same graph with the parent function. Describe the domain and range using set notation.

4. $g(x) = -3\sqrt{x-2} + 3$ **See margin.**

5. $g(x) = \sqrt{\frac{1}{3}(x+2)} + 1$ **See margin.**

Explain 2 Writing Square Root Functions

Given the graph of a square root function and the form of the transformed function, either $g(x) = a\sqrt{x-h} + k$ or $g(x) = \sqrt{\frac{1}{b}(x-h)} = k$, the transformation parameters can be determined from the transformed reference points. In either case, the initial point will be at (h, k) and readily apparent. The parameter a can be determined by how far up or down the second point (found at $x = h + 1$) is from the initial point, or the parameter b can be determined by how far to the left or right the second point (found at $y = k + 1$) is from the initial point.

Language Support EL

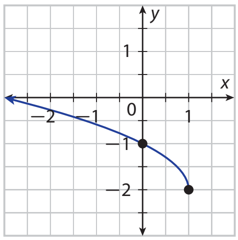
Communicate Math

Have students work in pairs to complete a chart similar to the following, comparing and contrasting quadratic and square root functions.

Type of Function	Example	Description	Similarities	Differences

Example 2 **Write the function that matches the graph using the indicated transformation format.**

(A) $g(x) = \sqrt{\frac{1}{b}(x - h)} + k$



Initial point: $(h, k) = (1, -2)$

Second point:

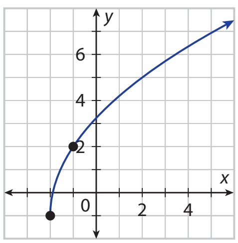
$(h + b, k + 1) = (0, -1)$

$1 + b = 0$

$b = -1$

The function is $g(x) = \sqrt{-1(x - 1)} - 2$.

(B) $g(x) = a\sqrt{x - h} + k$



Initial point: $(h, k) = \left(\boxed{-2}, \boxed{-1}\right)$

Second point:

$\left(h + 1, k + \boxed{a}\right) = \left(-1, \boxed{2}\right)$

$\boxed{-1} + a = 2$

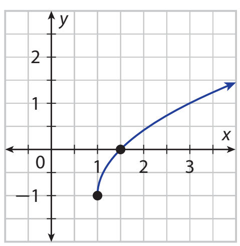
$a = \boxed{3}$

The function is $g(x) = \boxed{3}\sqrt{x \boxed{+} 2} - \boxed{1}$.

Your Turn

Write the function that matches the graph using the indicated transformation format.

6. $g(x) = \sqrt{\frac{1}{b}(x - h)} + k$ $\mathbf{g(x) = \sqrt{2(x - 1)} - 1}$



Integrate Technology

Students can use a graphing calculator to check their work. They can enter their functions and use the graphing feature to check that the graphs of their functions match the given graph.

Avoid Common Errors

Students may forget that when indicating a horizontal stretch or compression of the parent graph, the reciprocal of the stretch or compression factor, and not the factor itself, is placed under the radical sign. Present a side-by-side comparison of a stretch and a compression to remind students of the proper procedure.

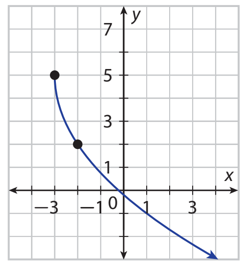
Explain 3

Modeling with Square Root Functions

Questioning Strategies

How can you summarize the rate of change of a square root function of the form $f(x) = a\sqrt{x}$ when a is a positive number? How is this reflected in the graph of the function? As values of x increase, the rate of change decreases. The graph shows a rising curve that gets less and less steep (i.e., flatter) as x approaches infinity.

7. $g(x) = a\sqrt{(x-h)} + k$ $\quad$ $g(x) = -3\sqrt{x+3} + 5$



Explain 3 Modeling with Square Root Functions

Square root functions that model real-world situations can be used to investigate average rates of change.

Recall that the average rate of change of the function $f(x)$ over an interval from x_1 to x_2 is given by

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1}.$$

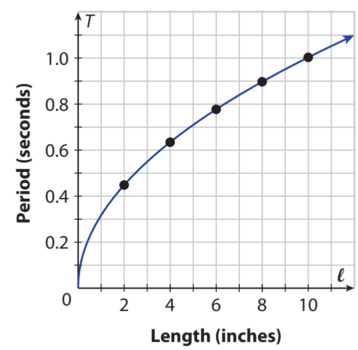
Example 3 **Use a calculator to evaluate the model at the indicated points, and connect the points with a curve to complete the graph of the model. Calculate the average rates of change over the first and last intervals and explain what the rate of change represents.**

(A) The approximate period T of a pendulum (the time it takes a pendulum to complete one swing) is given in seconds by the formula $T = 0.32\sqrt{\ell}$, where ℓ is the length of the pendulum in inches. Use lengths of 2, 4, 6, 8, and 10 inches.

First find the points for the given x-values.

Length (inches)	Period (seconds)
2	0.45
4	0.64
6	0.78
8	0.91
10	1.01

Plot the points and draw a smooth curve through them.



Find the average increase in period per inch increase in the pendulum length for the first interval and the last interval.

First interval:

$$\text{rate of change} = \frac{0.64 - 0.45}{4 - 2}$$
$$= 0.095$$

Last Interval:

$$\text{rate of change} = \frac{1.01 - 0.91}{10 - 8} = 0.05$$

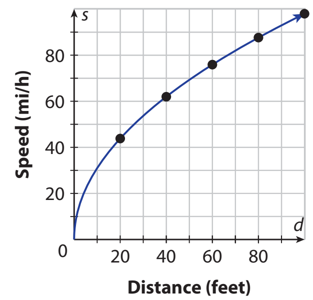
The average rate of change is less for the last interval. The average rate of change represents the increase in pendulum period with each additional inch of length. As the length of the pendulum increases, the increase in period time per inch of length becomes less.

(B) A car with good tires is on a dry road. The speed, in miles per hour, from which the car can stop in a given distance d, in feet, is given by $s(d) = \sqrt{96d}$. Use distances of 20, 40, 60, 80, and 100 feet.

First, find the points for the given x-values.

Distance	20	40	60	80	100
Speed	43.8	62.0	75.9	87.6	98.0

Plot the points and draw a smooth curve through them.



First interval:

$$\text{rate of change} = \frac{\boxed{62.0} - \boxed{43.8}}{40 - 20} = \boxed{0.91}$$

Last Interval:

$$\text{rate of change} = \frac{\boxed{98.0} - \boxed{87.6}}{100 - 80} = \boxed{0.52}$$

The average rate of change is less for the last interval. The average rate of change represents the increase in speed with each additional foot of distance. As the available stopping distance increases, the additional increase in speed per foot of stopping distance decreases.

Your Turn

Use a calculator to evaluate the model at the indicated points, and connect the points with a curve to complete the graph of the model. Calculate the average rates of change over the first and last intervals and explain what the rate of change represents.

8. The speed in miles per hour of a tsunami can be modeled by the function $s(d) = 3.86\sqrt{d}$, where d is the average depth in feet of the water over which the tsunami travels. Graph this function from depths of 1000 feet to 5000 feet and compare the change in speed with depth from the shallowest interval to the deepest. Use depths of 1000, 2000, 3000, 4000, and 5000 feet for the x-values. **See margin.**

Elaborate

9. What is the difference between the parameters inside the radical (b and h) and the parameters outside the radical (a and k)? **The inside parameters are horizontal transformations and the outside parameters are vertical transformations.**

10. Which transformations change the square root function's end behavior?

vertical reflections ($a < 0$) and horizontal reflections ($b < 0$)

Elaborate

Integrate Math Processes and Practices Focus on Problem Solving

MPP1 Enhance students' understanding of the different effects produced by a and b by having them complete a table of values for $x = 0, 1, 4, 16,$ and 64 and the functions $f(x) = \sqrt{x}$, $g(x) = 4\sqrt{x}$, $h(x) = \sqrt{4x}$, and $j(x) = \sqrt{\frac{1}{4}x}$, and compare the values across the functions. Students can then plot the points and graph the different functions to see how the effects of the parameters are reflected in the graphs.

Questioning Strategies

How can you use the rule for a square root function to determine the x-intercept of its graph? Find the value of x that makes the function equal to 0. This can be found by inspection, or by setting the rule equal to 0 and solving for x.

Summarize The Lesson

How is the graph of $f(x) = a\sqrt{x - h} + k$ related to the graph of $f(x) = \sqrt{x}$? Its starting point is at (h, k), instead of $(0, 0)$. It is stretched vertically by a factor of a if $|a| > 1$, or compressed vertically by a factor of a if $0 < |a| < 1$. If $a < 0$, then the graph is decreasing instead of increasing.

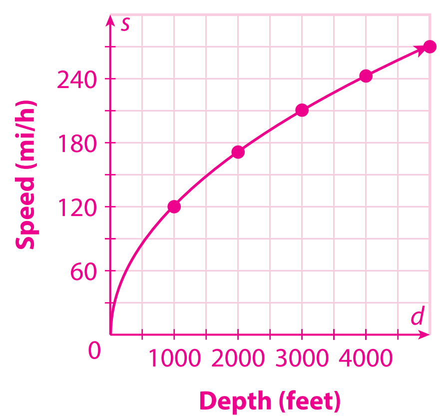
Answers

8. Points: (1000, 122.1), (2000, 172.6), (3000, 211.4), (4000, 244.1), (5000, 272.9)

$$\text{First interval:} = \frac{172.6 - 122.1}{2000 - 1000} = 0.0505$$

$$\text{Last Interval:} = \frac{272.9 - 244.1}{5000 - 4000} = 0.0288$$

The average rate of change is less for the last interval. The average rate of change represents the increase in tsunami speed with each additional foot of depth. The speed increases less with each additional foot of depth.



Evaluate

Assignment Guide

Level	Concepts and Skills	Practice
Basic	Explore 1	Exercise 1
	Explore 2	Exercises 2–3
	Example 1	Exercises 6–7, 10–11
	Example 2	Exercises 15–16
	Example 3	Exercise 20
	H.O.T.	Exercise 23
Average	Explore 1	Exercise 1
	Explore 2	Exercises 2–5
	Example 1	Exercises 6–8, 10–11
	Example 2	Exercises 14, 16–17
	Example 3	Exercises 18–19
	H.O.T.	Exercise 25
Advanced	Explore 1	N/A
	Explore 2	Exercises 4–5
	Example 1	Exercises 8–9, 12–13
	Example 2	Exercises 14, 17
	Example 3	Exercises 18–21
	H.O.T.	Exercises 23–25

Real World Problems

Integrate Math Processes and Practices

Focus on Using and Evaluating Logical Reasoning

MPP3 Mario says that the domain of the square root function is the set of all positive real numbers. Is he correct? Explain. He is not correct, because the domain also includes 0. The domain is the set of all non-negative real numbers.

Elaborate Answers

12. horizontal translations $(h \neq 0)$ and horizontal reflections $(a < 0)$

13. vertical translations $(k \neq 0)$ and vertical reflections $(b < 0)$

Evaluate

8. Domain: $\{x \mid x \leq -1\}$; Range: $\{y \mid y \geq 2\}$

11. horizontal translations $(h \neq 0)$ and vertical translations $(k \neq 0)$

11. Which transformations change the square root function's initial point location?

12. Which transformations change the square root function's domain? **12–13. See margin.**

13. Which transformations change the square root function's range?

14. **Essential Question Check-In** Describe in your own words the steps you would take to graph a function of the form $g(x) = a\sqrt{x-h} + k$ or $g(x) = \sqrt{\frac{1}{b}(x-h)} + k$ if you were given the values of h and k and using either a or b. **See Additional Answers.**

Evaluate: Homework and Practice

- Online Homework
- Hints and Help
- Extra Practice

1. Graph the functions $f(x) = \sqrt{x}$ and $g(x) = -\sqrt{x}$ on the same grid. Describe the domain, range and end behavior of each function. How are the functions related? **See Additional Answers.**

Describe the transformations of $g(x)$ from the parent function $f(x) = \sqrt{x}$.

2. $g(x) = \sqrt{\frac{1}{2}x} + 1$
Stretch horizontally by a factor of 2 and translate up by 1.

3. $g(x) = -5\sqrt{x+1} - 3$
3. Stretch vertically by a factor of 5, reflect across x-axis, and translate down by 3 and left by 1.

4. $g(x) = \frac{1}{4}\sqrt{x-5} - 2$
Compress vertically by a factor of $\frac{1}{4}$, and translate right by 5 and down by 2.

5. $g(x) = \sqrt{-7(x-7)}$
Compress horizontally by a factor of $\frac{1}{7}$, reflect across y-axis, and translate right by 7.

Describe the domain and range of each function using set notation.

6. $g(x) = \sqrt{\frac{1}{3}(x-1)}$ **Domain: $\{x \mid x \geq 1\}$ Range: $\{y \mid y \geq 0\}$**

7. $g(x) = 3\sqrt{x+4} + 3$ **7. Domain: $\{x \mid x \geq -4\}$ Range: $\{y \mid y \geq 3\}$**

8. $g(x) = \sqrt{-5(x+1)} + 2$ **See margin.**

9. $g(x) = -7\sqrt{x-3} - 5$ **Domain: $\{x \mid x \geq 3\}$ Range: $\{y \mid y \leq -5\}$**

Plot the transformed function $g(x)$ on a grid with the parent function, $f(x) = \sqrt{x}$. Describe the domain and range of each function using set notation.

10–13. Additional Answers.

10. $g(x) = -\sqrt{x} + 3$

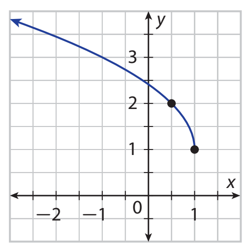
11. $g(x) = \sqrt{\frac{1}{3}(x+4)} - 1$

12. $g(x) = \sqrt{-\frac{2}{3}\left(x - \frac{1}{2}\right)} - 2$

13. $g(x) = 4\sqrt{x+3} - 4$

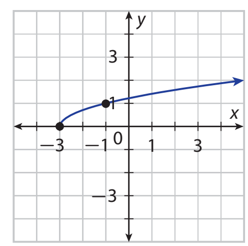
Write the function that matches the graph using the indicated transformation format.

14. $g(x) = \sqrt{\frac{1}{b}(x-h)} + k$



$g(x) = \sqrt{-2(x-1)} + 1$

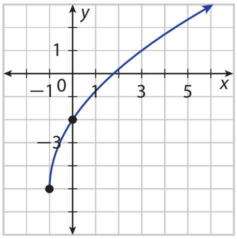
15. $g(x) = \sqrt{\frac{1}{b}(x-h)} + k$



$g(x) = \sqrt{\frac{1}{2}(x+3)}$

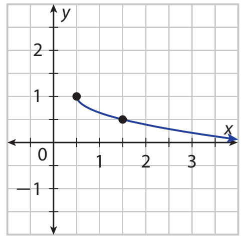
Exercise	Depth of Knowledge (D.O.K.)	Math Processes and Practices
1	**1** Recall of Information	**MPP6** Using Precise Mathematical Language
2–5	**2** Skills/Concepts	**MPP2** Abstract and Quantitative Reasoning
6–13	**2** Skills/Concepts	**MPP4** Mathematical Modeling
14–17	**2** Skills/Concepts	**MPP2** Abstract and Quantitative Reasoning
18–21	**2** Skills/Concepts	**MPP4** Mathematical Modeling
22	**2** Skills/Concepts	**MPP2** Abstract and Quantitative Reasoning

16. $g(x) = a\sqrt{x-h} + k$



$g(x) = 3\sqrt{x+1} - 5$

17. $g(x) = a\sqrt{x-h} + k$



$g(x) = -\frac{1}{2}\sqrt{x - \frac{1}{2}} + 1$

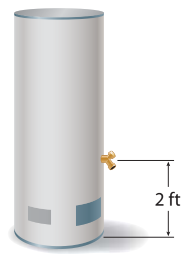
Use a calculator to evaluate the model at the indicated points, and connect the points with a curve to complete the graph of the model. Calculate the average rates of change over the first and last intervals and explain what the rate of change represents.

18–21. See Additional Answers.

18. A farmer is trying to determine how much fencing to buy to make a square holding pen with a 6-foot gap for a gate. The length of fencing, f, in feet, required as a function of area, A, in square feet, is given by $f(A) = 4\sqrt{A} - 6$. Evaluate the function from 20 ft² to 100 ft² by calculating points every 20 ft².

19. The speed, s, in feet per second, of an object dropped from a height, h, in feet, is given by the formula $s(h) = \sqrt{64h}$. Evaluate the function for heights of 0 feet to 25 feet by calculating points every 5 feet.

20. Water is draining from a tank at an average speed, s, in feet per second, characterized by the function $s(d) = 8\sqrt{d-2}$, where d is the depth of the water in the tank in feet. Evaluate the function for depths of 2, 3, 4, and 5 feet.



21. A research team studies the effects from an oil spill to develop new methods in oil clean-up. In the spill they are studying, the damaged oil tanker spilled oil into the ocean, forming a roughly circular spill pattern. The spill expanded out from the tanker, increasing the area at a rate of 100 square meters perhour. The radius of the circle is given by the function $r = \sqrt{\frac{100}{\pi}t}$, where t is the time (in hours) after the spill begins. Evaluate the function at hours 0, 1, 2, 3, and 4.

Exercise	Depth of Knowledge (D.O.K.)	Math Processes and Practices
23–24	**3** Strategic Thinking H.O.T.	**MPP2** Abstract and Quantitative Reasoning
25	**3** Strategic Thinking H.O.T.	**MPP4** Mathematical Modeling

Avoid Common Errors

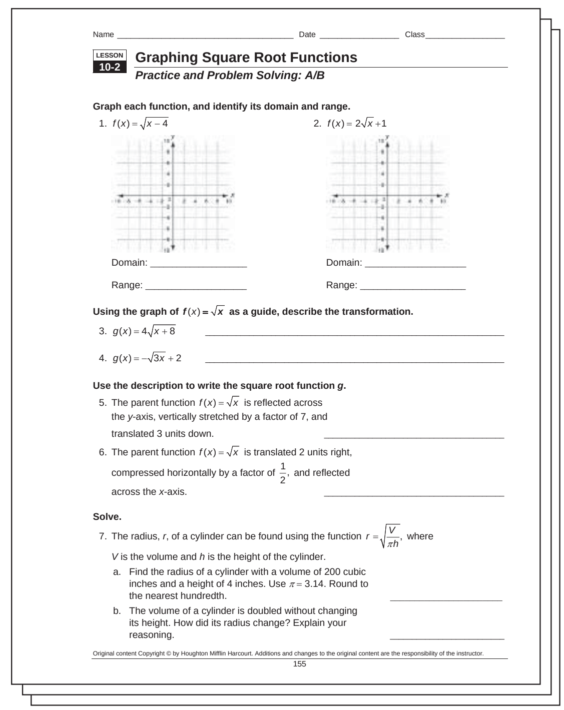
When graphing a function of the form $f(x) = a\sqrt{x-h} + k$ with $a < 0$, some students may perform the translation first and then perform the reflection. Help them to see that the function indicates a vertical translation of the graph of $f(x) = a\sqrt{x-h}$ by k units, so the graph needs to be reflected before it is translated vertically.

Journal

Have students explain how to find the equation of a transformed square root function given the starting point and another point on the graph.

Online Resources

- Practice and Problem Solving (three forms)
- Reteach
- Reading Strategies
- Success for English Learners


Name ______ Date ______ Class ______

LESSON 10-2 **Graphing Square Root Functions**
Practice and Problem Solving: A/B

Graph each function, and identify its domain and range.

1. $f(x) = \sqrt{x-4}$
2. $f(x) = 2\sqrt{x} + 1$

Domain: ______ Range: ______

Using the graph of $f(x) = \sqrt{x}$ as a guide, describe the transformation.

3. $g(x) = 4\sqrt{x+8}$
4. $g(x) = -\sqrt{3x} + 2$

Use the description to write the square root function g.

5. The parent function $f(x) = \sqrt{x}$ is reflected across the y-axis, vertically stretched by a factor of 7, and translated 3 units down.
6. The parent function $f(x) = \sqrt{x}$ is translated 2 units right, compressed horizontally by a factor of $\frac{1}{2}$, and reflected across the x-axis.

Solve.

7. The radius, r, of a cylinder can be found using the function $r = \sqrt{\frac{V}{\pi h}}$, where V is the volume and h is the height of the cylinder.
   a. Find the radius of a cylinder with a volume of 200 cubic inches and a height of 4 inches. Use $\pi = 3.14$. Round to the nearest hundredth.
   b. The volume of a cylinder is doubled without changing its height. How did its radius change? Explain your reasoning.

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Avoid Common Errors

Some students may think that the function $s(x) = \sqrt{x} + 1$ translates the function $f(x) = \sqrt{x}$ in the x direction because the $+1$ is on the same side of the equation as the variable x. Explain to students that $s(x) = \sqrt{x} + 1 = f(x) + 1$, which means that $s(x)$ is $f(x)$ increased by 1. Ask students to write the function $t(x)$ that translates $f(x)$ in the x direction one unit. $t(x) = \sqrt{x+1}$ or $t(x) = \sqrt{x-1}$

Questioning Strategies

What are the domain and range of the mathematical function $s(x) = \sqrt{x} + 1$? The domain is $x \geq 0$, and the range is $y \geq 1$. Both include continuous values.

Answers

24. $a\sqrt{x} = \sqrt{\frac{1}{b}x}$

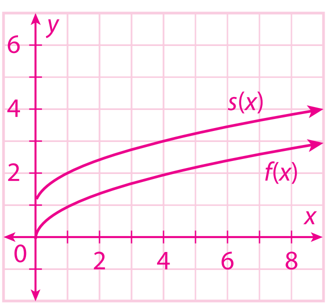
$a\sqrt{x} = \sqrt{\frac{1}{b}} \cdot \sqrt{x}$

$a = \sqrt{\frac{1}{b}}$

a and b must be positive numbers.

Lesson Performance Task

The function $s(x)$ is function $f(x)$ shifted 1 unit up.



If 45 containers of beans come in, then:

$s(x) = \sqrt{x} + 1$

$s(45) = \sqrt{45} + 1$

$s(45) \approx 8$

About 8 samples should be taken. The answer must be a whole number, because each container sampled produces one sample.

Lesson Performance Task Scoring Rubric

Points	Criteria
2	Student correctly solves the problem and explains his/her reasoning.
1	Student shows good understanding of the problem but does not fully solve or explain his/her reasoning.
0	Student does not demonstrate understanding of the problem.

22. Give all of the transformations of the parent function $f(x) = \sqrt{x}$ that result in the function $g(x) = \sqrt{-2(x-3)} + 2$. **B, C, D, and H**

A. Horizontal stretch
B. Horizontal compression
C. Horizontal reflection
D. Horizontal translation
E. Vertical stretch
F. Vertical compression
G. Vertical reflection
H. Vertical translation

H.O.T. Focus on Higher Order Thinking

23. Draw Conclusions Describe the transformations to $f(x) = \sqrt{x}$ that result in the function $g(x) = \sqrt{-8x + 16} + 3$.

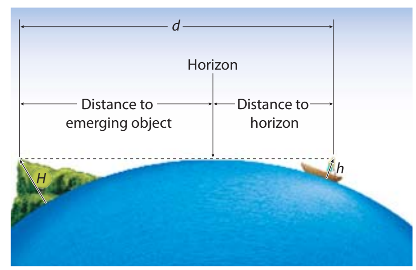
23. The transformation is a horizontal compression by a factor of $\frac{1}{8}$, a reflection across the y-axis, a translation of 2 to the right, and a translation of 3 up.

24. Analyze Relationships Show how a horizontally stretched square root function can sometimes be replaced by a vertical compression by equating the two forms of the transformed square root function.

$$g(x) = a\sqrt{x} = \sqrt{\frac{1}{b}x}$$

What must you assume about a and b for this replacement to result in the same function? **See margin.**

25. Multi-Step On a clear day, the view across the ocean is limited by the curvature of Earth. Objects appear to disappear below the horizon as they get farther from an observer. For an observer at height h above the water looking at an object with a height of H (both in feet), the approximate distance (d) in miles at which the object drops below the horizon is given by $d(h) = 1.21\sqrt{h+H}$. **See Additional Answers.**



a. What is the effect of the object height, H, on the graph of $d(h)$?

b. What is the domain of the function $d(h)$? Explain your answer.

c. Plot two functions of distance required to see an object over the horizon versus observer height: one for seeing a 2-foot-tall buoy and one for seeing a 20-foot-tall sailboat. Calculate points every 10 feet from 0 to 40 feet.

d. Where is the greatest increase in viewing distance with observer height?

Lesson Performance Task

With all the coffee beans that come in for processing, a coffee manufacturer cannot sample all of them. Suppose one manufacturer uses the function $s(x) = \sqrt{x} + 1$ to determine how many samples that it must take from x containers in order to obtain a good representative sampling of beans. How does this function relate to the function $f(x) = \sqrt{x}$? Graph both functions. How many samples should be taken from a shipment of 45 containers of beans? Explain why this can only be a whole number answer. **See margin.**

Extension Activity

If the manufacturer wants to increase the sample size to get a more accurate picture of the quality of the shipment, how can the model $s(x) = \sqrt{x} + 1$ be adjusted to increase the sample size? Apply a vertical stretching to get $s(x) = a\sqrt{x} + 1$, where $a > 1$, or increase the shift in the y direction to get $s(x) = \sqrt{x} + k$, where $k > 1$.

Ask students what might be a disadvantage of increasing the sample size. If the beans are destroyed during the sampling process, then the manufacturer has lost more of its inventory.