10.1 Inverses of Simple Quadratic and Cubic Functions

Essential Question: What functions are the inverses of quadratic functions and cubic functions, and how can you find them?

Resource Locker

Explore Finding the Inverse of a Many-to-One Function

The function $f(x)$ is defined by the following ordered pairs: $(-2, 4)$, $(-1, 2)$, $(0, 0)$, $(1, 2)$, and $(2, 4)$.

(A) Find the inverse function of $f(x)$, $f^{-1}(x)$, by reversing the coordinates in the ordered pairs. ?
(4, −2), (2, −1), (0, 0), (2, 1), (4, 2)

(B) Is the inverse also a function? Explain. ?
No; the *x*-values 2 and 4 are both paired with more than one *y*-value.

(C) If necessary, restrict the domain of $f(x)$ such that the inverse, $f^{-1}(x)$, is a function.

To restrict the domain of $f(x)$ so that its inverse is a function, you can restrict it to $\{x \mid x \geq \boxed{?}\}$. **0**

(D) With the restricted domain of $f(x)$, what ordered pairs define the inverse function $f^{-1}(x)$? ?
(0, 0), (2, 1), (4, 2)

Reflect

1. **Discussion** Look again at the ordered pairs that define $f(x)$. Without reversing the order of the coordinates, how could you have known that the inverse of $f(x)$ would not be a function? **See below.**

2. How will restricting the domain of $f(x)$ affect the range of its inverse? **See margin.**

Explain 1 Finding and Graphing the Inverse of a Simple Quadratic Function

The function $f(x) = x^2$ is a many-to-one function, so its domain must be restricted in order to find its inverse function. If the domain is restricted to $x \geq 0$, then the inverse function is $f^{-1}(x) = \sqrt{x}$; if the domain is restricted to $x \leq 0$, then the inverse function is $f^{-1}(x) = -\sqrt{x}$.

The inverse of a quadratic function is a **square root function**, which is a function whose rule involves $\sqrt{x}$. **The parent square root function** is $g(x) = \sqrt{x}$. A square root function is defined only for values of x that make the expression under the radical sign nonnegative.

1. When two or more ordered pairs have the same *y*-value but unique *x*-values, the inverse of that function will have coordinates in which the same *x*-value maps to more than one *y*-value, and thus will not be a function. That is, if a function is many-to-one, its inverse will not be a function.

Professional Development

Math Background

The domain of any even function $f(x) = x^n$ where $n = 2, 4, 6, \ldots$ needs to be restricted for the inverse to also be a function. This is because these functions are not one-to-one. If the domain is not restricted, then reflecting its graph across the line $y = x$ yields a graph that fails the vertical line test. This is because every non-zero real number has two nth roots when n is even, one positive and one negative. In contrast, the domain of any odd function $f(x) = x^n$ where $n = 1, 3, 5, \ldots$ need not be restricted for the inverse to also be a function. This is because every real number has a single nth root when n is odd, either positive, negative, or zero at the origin.

Inverses of Simple Quadratic and Cubic Functions

Learning Objective

Students will find inverse functions for simple quadratic and cubic functions, restricting domains when needed.

Math Processes and Practices

MPP2 Abstract and Quantitative Reasoning

Language Objective

Fill in an organizer of quadratic and cubic functions and their inverses.

Online Resources

An extra example for each Explain section is available online.

Engage

Essential Question: What functions are the inverses of quadratic and cubic functions, and how can you find them?

Possible answer: Square root functions are the inverses of quadratic functions, with the included restricted domain. Cube root functions are the inverses of cubic functions. You can find the inverse functions by using inverse operations and switching the variables, but must restrict the domain of a quadratic function.

Preview: Lesson Performance Task

View the Engage section online. Discuss the photo and how the irrigation area can be described by a quadratic function. Ask students to list the variables involved in the function. Then preview the Lesson Performance Task.

Answer to Reflect 2. See page 346.

Explore

Finding the Inverse of a Many-to-One Function

Integrate Technology

Students can use the DrawInv capability of their graphing calculators to draw the inverse of the graph of this function and others, but they should realize that the calculator will draw the inverse whether or not the inverse is also a function.

Questioning Strategies

How can you tell if a set of ordered pairs that represents a function has an inverse that is a function? Check to see whether each ordered pair has a unique *y*-value.

Explain 1

Finding and Graphing the Inverse of a Simple Quadratic Function

Questioning Strategies

What characteristic of the graph of a quadratic function tells you that the domain needs to be restricted for the inverse to be a function? The U-shape tells you that the function is not one-to-one, and that its inverse will not be a function unless the domain is restricted.

How do you find the domain and range of the inverse of a quadratic function? The domain will be the range of the original quadratic function. The range will be the domain of the original quadratic function, restricted to values either greater than or equal to, or less than or equal to, the value of *x* that produces the minimum or maximum value of the original function.

Answer to Reflect 2

2. Since the domain of a function is the same as the range of its inverse, restricting the domain of $f(x)$ restricts the range of its inverse.

Example 1 **Restrict the domain of each quadratic function and find its inverse. Confirm the inverse relationship using composition. Graph the function and its inverse.**

A $f(x) = 0.5x^2$

Restrict the domain. $\{x \mid x \geq 0\}$

Find the inverse.

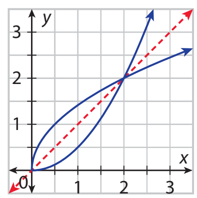
Replace $f(x)$ with y.	$y = 0.5x^2$
Multiply both sides by 2.	$2y = x^2$
Use the definition of positive square root.	$\sqrt{2y} = x$
Switch x and y to write the inverse.	$\sqrt{2x} = y$
Replace y with $f^{-1}(x)$.	$f^{-1}(x) = \sqrt{2x}$

Confirm the inverse relationship using composition.

$$\begin{aligned} f^{-1}(f(x)) &= f^{-1}(0.5x^2) \\ &= \sqrt{2(0.5x^2)} \\ &= \sqrt{x^2} \\ &= x \text{ for } x \geq 0 \end{aligned} \qquad \begin{aligned} f(f^{-1}(x)) &= 0.5(\sqrt{2x})^2 \\ &= 0.5(2x) \\ &= x \text{ for } x \geq 0 \end{aligned}$$

Since $f^{-1}(f(x)) = x$ for $x \geq 0$ and $f(f^{-1}(x)) = x$ for $x \geq 0$, it has been confirmed that $f^{-1}(x) = \sqrt{2x}$ for $x \geq 0$ is the inverse function of $f(x) = 0.5x^2$ for $x \geq 0$.

Graph $f^{-1}(x)$ by graphing $f(x)$ over the restricted domain and reflecting the graph over the line $y = x$.



B $f(x) = x^2 - 7$

Find the inverse.

Restrict the domain. $\{x \mid x \geq 0\}$

Replace $f(x)$ with y.	$\boxed{y} = x^2 - 7$
Add 7 to both sides.	$\boxed{y + 7} = x^2$
Use the definition of positive square root.	$\boxed{\sqrt{x + 7}} = x$

Switch x and y to write the inverse. $\sqrt{x+7} = y$

Replace y with $f^{-1}(x)$. $\sqrt{x+7} = f^{-1}(x)$

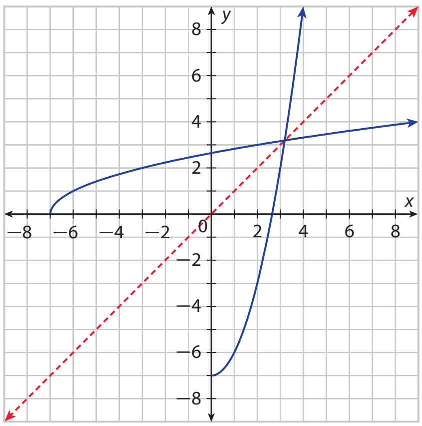
Confirm the inverse relationship using composition.

$$\begin{aligned} f^{-1}(f(x)) &= f^{-1}\left(\boxed{x^2 - 7}\right) \\ &= \boxed{\sqrt{(x^2-7)+7}} \\ &= \boxed{\sqrt{x^2}} \\ &= \boxed{x \text{ for } x \geq 0} \end{aligned}$$

$$\begin{aligned} f(f^{-1}(x)) &= f\left(\boxed{\sqrt{x+7}}\right) \\ &\left(\boxed{\sqrt{x+7}}\right)^2 - 7 \\ &= \boxed{x+7} - 7 \\ &= \boxed{x \geq -7} \end{aligned}$$

Since $f^{-1}(f(x)) = \boxed{x}$ for $x \geq 0$ it has been confirmed that $f^{-1}(x) = \boxed{\sqrt{x+7}}$ for $\boxed{x \geq -7}$ is the inverse function of $f(x) = x^2 - 7$ for $x \geq 0$.

Graph $f^{-1}(x)$ by graphing $f(x)$ over the restricted domain and reflecting the graph over the line $y = x$.



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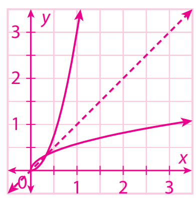


Your Turn

Restrict the domain of the quadratic function and find its inverse. Confirm the inverse relationship using composition. Graph the function and its inverse.

3. $f(x) = 3x^2$ **Restrict the domain:** $\{x \mid x \geq 0\}$. **Find the inverse:** $f^{-1}(x) = \sqrt{\frac{x}{3}}$

Explain 2 Finding the Inverse of a Quadratic Model

In many instances, quadratic functions are used to model real-world applications. It is often useful to find and interpret the inverse of a quadratic model. Note that when working with real-world applications, it is more useful to use the notation $x(y)$ for the inverse of $y(x)$ instead of the notation $y^{-1}(x)$.

Integrate Math Processes and Practices

Focus on Problem Solving

MPP1 Remind students how the vertical line test can be used to determine whether the graph of a relation represents a function. Help them see that the horizontal line test can be used to see whether a function has an inverse that is a function. Lead them to recognize how applying the horizontal line test to the graph of the original function is related to applying the vertical line test to the graph of the inverse of the function.

Avoid Common Errors

Students may make algebraic errors when finding the inverse of a function such as $f(x) = 5x^2$. Reinforce that students should isolate x^2 before taking the square root of each side, and that when taking the square root, they must take the square root of the entire expression on the other side of the equation, not just of the variable.

Explain 2

Finding the Inverse of a Quadratic Model

Questioning Strategies

How does the inverse of a function relate to the original function in a real-world application? It tells you how to find the value of what was the independent variable in the original function, given the value of what was the dependent variable.

Integrate Math Processes and Practices
Focus on Mathematical Modeling

MPP4 Discuss with students how a function and its inverse can both model a given real-world situation, even if one is a power function and the other is a radical function.

Example 2 **Find the inverse of each of the quadratic functions. Use the inverse to solve the application.**

(A) The function $d(t) = 16t^2$ gives the distance d in feet that a dropped object falls in t seconds. Write the inverse function $t(d)$ to find the time t in seconds it takes for an object to fall a distance of d feet. Then estimate how long it will take a penny dropped into a well to fall 48 feet.

The original function $d(t) = 16t^2$ is a quadratic function with a domain restricted to $t \geq 0$.

Find the inverse function.

Write $d(t)$ as d. $\qquad d = 16t^2$

Divide both sides by 16. $\qquad \dfrac{d}{16} = t^2$

Use the definition of positive square root. $\qquad \sqrt{\dfrac{d}{16}} = t$

Write t as $t(d)$. $\qquad \sqrt{\dfrac{d}{16}} = t(d)$

The inverse function is $t(d) = \sqrt{\dfrac{d}{16}}$ for $d \geq 0$.

Use the inverse function to estimate how long it will take a penny dropped into a well to fall 48 feet. Substitute $d = 48$ into the inverse function.

Write the function. $\qquad t(d) = \sqrt{\dfrac{d}{16}}$

Substitute 48 for d. $\qquad t(48) = \sqrt{\dfrac{48}{16}}$

Simplify. $\qquad t(48) = \sqrt{3}$

Use a calculator to estimate. $\qquad t(48) \approx 1.7$

So, it will take about 1.7 seconds for a penny to fall 48 feet into the well.

(B) The function $E(v) = 4v^2$ gives the kinetic energy E in Joules of an 8-kg object that is traveling at a velocity of v meters per second. Write and graph the inverse function $v(E)$ to find the velocity v in meters per second required for an 8-kg object to have a kinetic energy of E Joules. Then estimate the velocity required for an 8-kg object to have a kinetic energy of 60 Joules.

The original function $E(v) = 4v^2$ is a quadratic function with a domain restricted to $v \geq 0$.

Find the inverse function.

Write $E(v)$ as E. $\qquad \boxed{E} = 4v^2$

Divide both sides by 4. $\qquad \boxed{\dfrac{E}{4}} = v^2$

Use the definition of positive square root. $\qquad \boxed{\sqrt{\dfrac{E}{4}}} = v$

Write v as $v(E)$. $v(E) = \sqrt{\frac{E}{4}}$

The inverse function is $v(E) = \sqrt{\frac{E}{4}}$ for $E \geq 0$.

Use the inverse function to estimate the velocity required for an 8-kg object to have a kinetic energy of 60 Joules.

Substitute $E = 60$ into the inverse function.

Write the function. $v(E) = \boxed{\sqrt{\frac{E}{4}}}$

Substitute 60 for E. $v\left(\boxed{60}\right) = \boxed{\sqrt{\frac{60}{4}}}$

Simplify. $v(60) = \sqrt{15}$

Use a calculator to estimate. $v(60) \approx 3.9$

So, an 8-kg object with kinetic energy of 60 Joules is traveling at a velocity of 3.9 meters per second.

Your Turn

Find the inverse of the quadratic function. Use the inverse to solve the application.

4. The function $A(r) = \pi r^2$ gives the area of a circular object with respect to its radius r. Write the inverse function $r(A)$ to find the radius r required for area of A. Then estimate the radius of a circular object that has an area of 40 cm^2. **$r(A) = \sqrt{\frac{A}{\pi}}$; A circular object with an area of 40 cm^2 will have a radius of about 3.6 cm.**

Explain 3 Finding and Graphing the Inverse of a Simple Cubic Function

Note that the function $f(x) = x^3$ is a one-to-one function, so its domain does not need to be restricted in order to find its inverse function. The inverse of $f(x) = x^3$ is $f^{-1}(x) = \sqrt[3]{x}$.

The inverse of a cubic function is a **cube root function**, which is a function whose rule involves $\sqrt[3]{x}$. The **parent cube root function** is $g(x) = \sqrt[3]{x}$.

Example 3 **Find the inverse of each cubic function. Confirm the inverse relationship using composition. Graph the function and its inverse.**

(A) $f(x) = 0.5x^3$

Find each inverse. Graph the function and its inverse.

Replace $f(x)$ with y. $y = 0.5x^3$

Multiply both sides by 2. $2y = x^3$

Use the definition of cube root. $\sqrt[3]{2y} = x$

Switch x and y to write the inverse. $\sqrt[3]{2x} = y$

Replace y with $f^{-1}(x)$. $\sqrt[3]{2x} = f^{-1}(x)$

Integrate Technology

A graphing calculator can be used to verify that functions are inverses of each other. Students can graph the two functions and the function $f(x) = x$, and check that the graphs of their functions are reflections of each other across the graph of $f(x) = x$.

Explain 3

Finding and Graphing the Inverse of a Simple Cubic Function

Questioning Strategies

Why is there no need to restrict the domain of a simple cubic function before finding its inverse? Because it is a one-to-one function, its inverse is also a function.

How does the domain of a cube root function compare with the domain of a square root function? Why? The domain of a cube root function is all real numbers. The domain of a square root function is the values of x that make the expression inside the square root greater than or equal to zero. The square root of a negative number is not a real number, but the cube root of a negative number is a real number.

Collaborative Learning

Peer-to-Peer Activity

Have students work in pairs. Provide each pair with a function of the form $f(x) = ax^3 + c$. Have students find $f^{-1}(x)$, the inverse of the function. Then have them graph both $f(x)$ and $f^{-1}(x)$ on the same coordinate grid. Have them check their work by verifying that if (x, y) belongs to $f(x)$, then (y, x) belongs to $f^{-1}(x)$.

Multiple Representations

Discuss with students how symbolic representations, graphs, and tables of values can all be used to analyze the relationship between a quadratic or cubic function and its inverse. Help students to make connections among the various representations. Different learners may find that one type of representation is more useful than others in aiding their understanding.

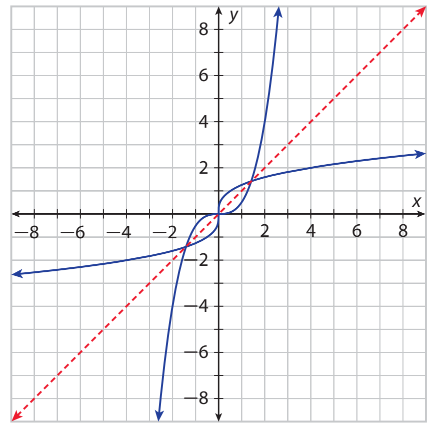
Confirm the inverse relationship using composition.

$$f^{-1}\big(f(x)\big) = f^{-1}(0.5x^3) = \sqrt[3]{2(0.5x^3)} = \sqrt[3]{x^3} = x$$

$$f\big(f^{-1}(x)\big) = f\big(\sqrt[3]{2x}\big) = 0.5\big(\sqrt[3]{2x}\big)^3 = 0.5(2x) = x$$

Since $f^{-1}\big(f(x)\big) = x$ and $f\big(f^{-1}(x)\big) = x$, it has been confirmed that $f^{-1}(x) = \sqrt[3]{2x}$ is the inverse function of $f(x) = 0.5x^3$.

Graph $f^{-1}(x)$ by graphing $f(x)$ and reflecting $f(x)$ over the line $y = x$.



(B) $f(x) = x^3 - 9$

Find the inverse.

Replace $f(x)$ with y.	$\boxed{y} = x^3 - 9$
Add 9 to both sides.	$\boxed{y + 9} = x^3$
Use the definition of cube root.	$\boxed{\sqrt[3]{y+9}} = x$
Switch x and y to write the inverse.	$\sqrt[3]{x+9} = y$
Replace y with $f^{-1}(x)$.	$\sqrt[3]{x+9} = f^{-1}(x)$

Confirm the inverse relationship using composition.

$$f^{-1}\big(f(x)\big) = f^{-1}\big(\boxed{x^3 - 9}\big) = \boxed{\sqrt[3]{(x^3-9)+9}} = \boxed{\sqrt[3]{x^3}} = \boxed{x}$$

$$f\big(f^{-1}(x)\big) = f\big(\boxed{\sqrt[3]{y+9}}\big) = \boxed{\big(\sqrt[3]{y+9}\big)^3 - 9} = \boxed{x + 9 - 9} = \boxed{x}$$

Differentiate Instruction

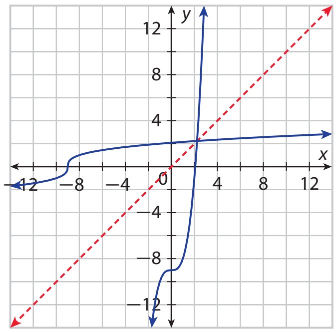
Graphic Organizers

Students may benefit from summarizing what they've learned in a graphic organizer like the one shown below. Encourage students to add graphs.

	Linear	Quadratic	Cubic
Functions	$f(x) = ax$ $f^{-1}(x) = \frac{x}{a}$	$f(x) = ax^2$ $f^{-1}(x) = \sqrt{\frac{x}{a}}$	$f(x) = ax^3$ $f^{-1}x = \sqrt[3]{\frac{x}{a}}$
Restrictions	$f(x)$ must have nonzero slope	Domain of $f(x)$: nonnegative values	No restrictions

Since $f^{-1}(f(x)) = \boxed{x}$ and $f(f^{-1}(x)) = \boxed{x}$, it has been confirmed that $f^{-1}(x) = \boxed{\sqrt[3]{x+9}}$ is the inverse function of $f(x) = x^3 - 9$.

Graph $f^{-1}(x)$ by graphing $f(x)$ and reflecting $f(x)$ over the line $y = x$.

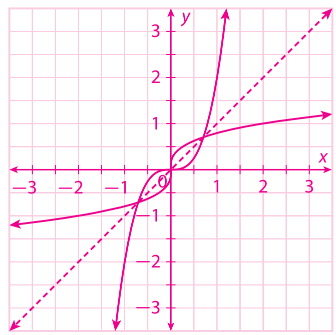


Your Turn

Find the inverse. Graph the function and its inverse.

5. $f(x) = 2x^3$

5. $f^{-1}(x) = \sqrt[3]{\dfrac{x}{2}}$



Explain 4 Finding the Inverse of a Cubic Model

In many instances, cubic functions are used to model real-world applications. It is often useful to find and interpret the inverse of cubic models. As with quadratic real-world applications, it is more useful to use the notation $x(y)$ for the inverse of $y(x)$ instead of the notation $y^{-1}(x)$.

Example 4 **Find the inverse of each of the following cubic functions.**

A) The function $m(L) = 0.00001L^3$ gives the mass m in kilograms of a red snapper of length L centimeters. Find the inverse function $L(m)$ to find the length L in centimeters of a red snapper that has a mass of m kilograms.

The original function $m(L) = 0.00001L^3$ is a cubic function.

Find the inverse function.

Write $m(L)$ as m.	$m = 0.00001L^3$
Multiply both sides by 100,000.	$100{,}000m = L^3$
Use the definition of cube root.	$\sqrt[3]{100{,}000m} = L$
Write L as $L(m)$.	$\sqrt[3]{100{,}000m} = L(m)$

The inverse function is $L(m) = \sqrt[3]{100{,}000m}$.

Integrate Math Processes and Practices
Focus on Using Mathematical Tools

MPP5 The table feature on a graphing calculator can be used to verify that two functions are inverses of each other. Students can enter both functions, and use the table to verify that if (x, y) belongs to Y1, then (y, x) belongs to Y2.

Explain 4

Finding the Inverse of a Cubic Model

Questioning Strategies

How do you determine the domain and range of the inverse of the function in a real-world application? The domain is the range of the original function, and the range is the domain of the original function. Domains may need to be restricted to values that are reasonable values of the independent variable in the given context.

Integrate Math Processes and Practices
Focus on Using and Evaluating Logical Reasoning

MPP3 To ensure that students understand the relationship between the function and its inverse in this context, ask them to describe real-world questions that could more easily be answered using the original function, and those that could more easily be answered using the inverse function.

Language Support EL

Communicate Math

Have students work in pairs. Ask them to discuss how to fill in a chart like the one below and, when they reach agreement about an entry, to take turns adding it to the chart.

Type of Function	Example equation	Graph of function
Quadratic function		
Inverse:		
Cubic function		
Inverse:		

Elaborate

Integrate Math Processes and Practices

Focus on Using and Evaluating Logical Reasoning

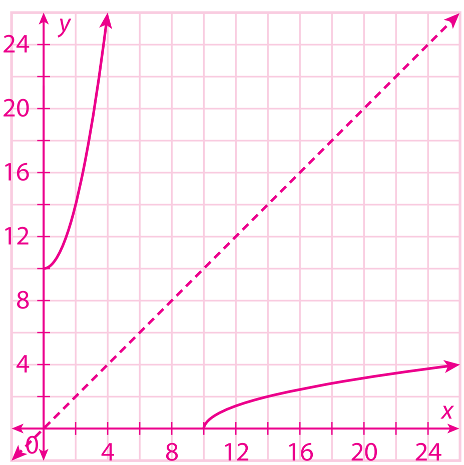
MPP3 Ask students to consider whether the end behavior of the graph of a function can be used to determine the need to restrict the domain in order for the inverse to be a function. Lead them to recognize that when the end behavior is the same as x approaches either ∞ or $-\infty$ (as is the case with a quadratic function), it *is* an indicator. However, when this is not the case, it is not possible to tell whether the function is one-to-one. Have students consider, for example, the graph of a cubic function that has a local maximum and a local minimum. This function is not one-to-one, but other cubic functions, such as $f(x) = x^3$, are.

Summarize The Lesson

How do you find the function that is the inverse of a quadratic function? What type of function is the inverse? Substitute y for $f(x)$, solve for x, and then switch x and y. The result is a square root function. You must restrict the domain of the original function so that the inverse will be a function.

Answers

3. Restrict the domain to $\{x \mid x \geq 0\}$.
$f^{-1}(x) = \sqrt{x - 10}$



B The function $V(r) = \frac{4}{3}\pi r^3$ gives the volume V of a sphere with radius r. Find the inverse function $r(V)$ to find the radius r of a sphere with volume V.

The original function $V(r) = \frac{4}{3}\pi r^3$ is a cubic function.

Find the inverse function.

Write $V(r)$ as V. $\boxed{V} = \frac{4}{3}\pi r^3$

Divide both sides by $\frac{4}{3}\pi$. $\boxed{\frac{3}{4\pi}V} = r^3$

Use the definition of cube root. $\boxed{\sqrt[3]{\frac{3}{4\pi}V}} = r$

Write r as $r(V)$. $\sqrt[3]{\frac{3}{4\pi}V} = r(V)$

The inverse function is $r(V) = \sqrt[3]{\frac{3}{4\pi}V}$.

9. The inverse function of a quadratic function is a square root function, which is not defined for nonnegative values of x, whereas the inverse function of a cubic function is a cube root function, which is defined for all real number values of x.

Your Turn **6.** $r(m) = \sqrt[3]{\frac{3}{44\pi}m}$

6. The function $m(r) = \frac{44}{3}\pi r^3$ gives the mass in grams of a spherical lead ball with a radius of r centimeters. Find the inverse function $r(m)$ to find the radius r of a lead sphere with mass m.

7. The inverse of $f(x) = ax^2$ is $f^{-1}(x) = \sqrt{\frac{x}{a}}$, where the domain of both functions must be restricted to $\{x \mid x \geq 0\}$.

Elaborate

7. What is the general form of the inverse function for the function $f(x) = ax^2$? State any restrictions on the domains.

8. What is the general form of the inverse function for the function $f(x) = ax^3$? State any restrictions on the domains. **The inverse of $f(x) = ax^3$ is $f^{-1}(x) = \sqrt[3]{\frac{x}{a}}$. There are no restrictions on the domains.**

9. **Essential Question Check-In** Why must the domain be restricted when finding the inverse of a quadratic function, but not when finding the inverse of a cubic function? **See above.**

Evaluate: Homework and Practice

Personal Math Trainer
• Online Homework
• Hints and Help
• Extra Practice

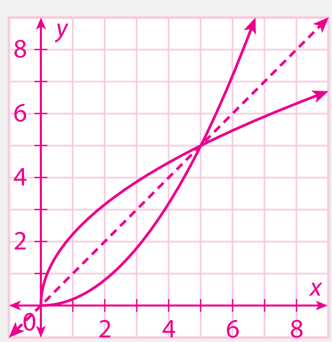
Restrict the domain of the quadratic function and find its inverse. Confirm the inverse relationship using composition. Graph the function and its inverse. **1–3. See margin.**

1. $f(x) = 0.2x^2$
2. $f(x) = 8x^2$
3. $f(x) = x^2 + 10$

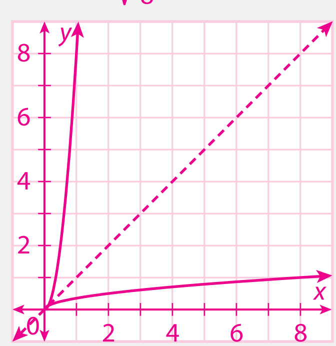
Restrict the domain of the quadratic function and find its inverse. Confirm the inverse relationship using composition.

4. $f(x) = 15x^2$ **Restrict the domain to $\{x \mid x \geq 0\}$. $f^{-1}(x) = \sqrt{\frac{x}{15}}$**
5. $f(x) = x^2 - \frac{3}{4}$ **Restrict the domain to $\{x \mid x \geq 0\}$. $f^{-1}(x) = \sqrt{x + \frac{3}{4}}$**
6. $f(x) = 0.7x^2$ **Restrict the domain to $\{x \mid x \geq 0\}$. $f^{-1} = (x)\sqrt{\frac{10}{7}x}$**

1. Restrict the domain to $\{x \mid x \geq 0\}$.
$f^{-1}(x) = \sqrt{5x}$



2. Restrict the domain to $\{x \mid x \geq 0\}$.
$f^{-1}(x) = \sqrt{\frac{x}{8}}$



7. The function $d(s) = \frac{1}{14.9}s^2$ models the average depth d in feet of the water over which a tsunami travels, where s is the speed in miles per hour. Write the inverse function $s(d)$ to find the speed required for a depth of d feet. Then estimate the speed of a tsunami over water with an average depth of 1500 feet.

$s(d) = \sqrt{14.9d}$

The speed of a tsunami over water with an average depth of 1500 feet is about 150 mi/h.

8. The function $x(T) = 9.8\left(\frac{T}{2\pi}\right)^2$ gives the length x in meters for a pendulum to swing for a period of T seconds. Write the inverse function to find the period of a pendulum in seconds. The period of a pendulum is the time it takes the pendulum to complete one back-and-forth swing. Find the period of a pendulum with length of 5 meters.

$T(x) = 2\pi\sqrt{\frac{x}{9.8}}$

The period of a pendulum with a length of 5 meters is about 4.5 seconds.

Find the inverse of each cubic function. Confirm the inverse relationship using composition. Graph the function and its inverse. **9–10. See Additional Answers for graphs.**

9. $f(x) = 0.25x^3$ $\quad f^{-1}(x) = \sqrt[3]{4x}$

10. $f(x) = -12x^3$ $\quad f^{-1}(x) = \sqrt[3]{-\frac{x}{12}}$

Find the inverse of the cubic function. Confirm the inverse relationship using composition.

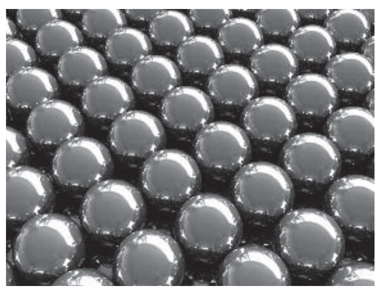
11. $f(x) = x^3 - \frac{5}{6}$ $\quad f^{-1}(x) = \sqrt[3]{x + \frac{5}{6}}$

12. $f(x) = x^3 + 9$ $\quad f^{-1}(x) = \sqrt[3]{x - 9}$

13. The function $m(r) = 31r^3$ models the mass in grams of a spherical zinc ball as a function of the ball's radius in centimeters. Write the inverse model to represent the radius r in cm of a spherical zinc ball as a function of the ball's mass m in g. $\quad r(m) = \sqrt[3]{\frac{m}{31}}$

14. The function $m(r) = 21r^3$ models the mass in grams of a spherical titanium ball as a function of the ball's radius in centimeters. Write the inverse model to represent the radius r in centimeters of a spherical titanium ball as a function of the ball's mass m in grams.

$r(m) = \sqrt[3]{\frac{m}{21}}$



Exercise	Depth of Knowledge (D.O.K.)	Math Processes and Practices
1–6	2 Skills/Concepts	MPP3 Using and Evaluating Logical Reasoning
7–8	3 Strategic Thinking	MPP4 Mathematical Modeling
9–10	2 Skills/Concepts	MPP6 Using Precise Mathematical Language
11–12	2 Skills/Concepts	MPP2 Abstract and Quantitative Reasoning
13–15	3 Strategic Thinking	MPP4 Mathematical Modeling
16,18	3 Strategic Thinking H.O.T.	MPP2 Abstract and Quantitative Reasoning
17	3 Strategic Thinking H.O.T.	MPP4 Mathematical Modeling

Evaluate

Personal Math Trainer

Assignment Guide

Level	Concepts and Skills	Practice
Basic	Example 1	Exercises 1–3
	Example 2	Exercises 7–8
	Example 3	Exercises 9, 11
	Example 4	Exercise 13
	H.O.T.	Exercise 16
Average	Example 1	Exercises 4–6
	Example 2	Exercises 7–8
	Example 3	Exercises 9–12
	Example 4	Exercise 14
	H.O.T.	Exercises 16–17
Advanced	Example 1	Exercises 4–6
	Example 2	Exercises 7–8
	Example 3	Exercises 10, 12
	Example 4	Exercises 14–15
	H.O.T.	Exercises 16–18

Real World Problems

Questioning Strategies

Why is it important to know whether a function is one-to-one when finding its inverse? Because if it is not one-to-one, you need to restrict the domain for the inverse to be a function.

Avoid Common Errors

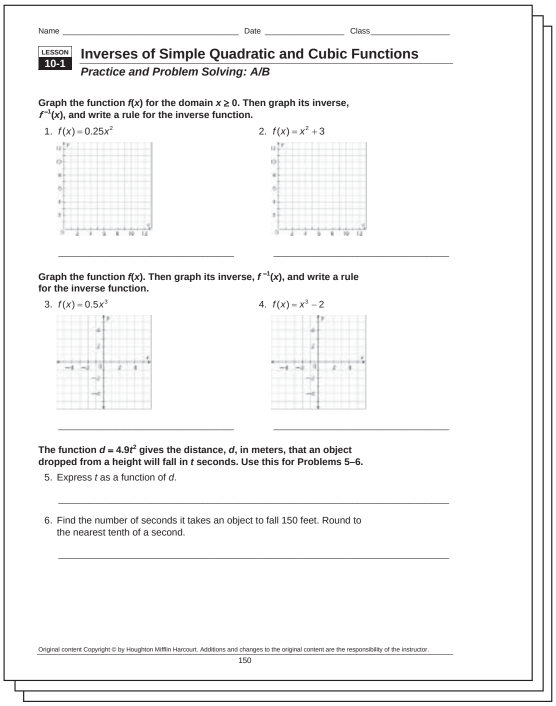
Students may, in error, believe that if the graph of a relation passes the horizontal line test, then the relation is one-to-one. Help them to see that the horizontal line test can be used only to test whether a *function* is one-to-one. In other words, the graph must first pass the vertical line test.

Journal

Have students describe the functions that are inverses of quadratic functions and cubic functions. Have them explain why it is sometimes necessary to restrict the domain of a function when finding its inverse.

Online Resources

- Practice and Problem Solving (three forms)
- Reteach
- Reading Strategies
- Success for English Learners



Name __________ Date __________ Class__________

LESSON 10-1 **Inverses of Simple Quadratic and Cubic Functions**

Practice and Problem Solving: A/B

Graph the function $f(x)$ for the domain $x \geq 0$. Then graph its inverse, $f^{-1}(x)$, and write a rule for the inverse function.

1. $f(x) = 0.25x^2$
2. $f(x) = x^2 + 3$

Graph the function $f(x)$. Then graph its inverse, $f^{-1}(x)$, and write a rule for the inverse function.

3. $f(x) = 0.5x^3$
4. $f(x) = x^3 - 2$

The function $d = 4.9t^2$ gives the distance, d, in meters, that an object dropped from a height will fall in t seconds. Use this for Problems 5–6.

5. Express t as a function of d.
6. Find the number of seconds it takes an object to fall 150 feet. Round to the nearest tenth of a second.

150

Lesson Performance Task Scoring Rubric

Points	Criteria
2	Student correctly solves the problem and explains his/her reasoning.
1	Student shows good understanding of the problem but does not fully solve or explain his/her reasoning.
0	Student does not demonstrate understanding of the problem.

15. After an initial deposit of \$2000, a bank account pays interest at an annual rate $r\%$ compounded annually. The value V of the account after 3 years can be represented by the model $V(r) = 2000\,(1 + r)^3$. Write the inverse function $r(V)$ to find the interest rate needed for the account to have value V after the 3 years. $\sqrt[3]{\dfrac{V}{2000}} - 1 = r(V)$

H.O.T. Focus on Higher Order Thinking

16. Explain the Error A student was asked to find the inverse of the function $f(x) = \left(\frac{x}{2}\right)^3 + 9$. What did the student do wrong? Find the correct inverse.

$$f(x) = \left(\frac{x}{2}\right)^3 + 9$$
$$y = \left(\frac{x}{2}\right)^3 + 9$$
$$y - 9 = \left(\frac{x}{2}\right)^3$$
$$2y - 18 = x^3$$
$$\sqrt[3]{2y - 18} = x$$
$$y = \sqrt[3]{2x - 18}$$
$$f^{-1}(x) = \sqrt[3]{2x - 18}$$

The student multiplied both sides by 2 before taking the cube root of both sides. The student should have taken the cube root of both sides first, and then multiplied by 2.

$f^{-1}(x) = 2\sqrt[3]{x - 9}$

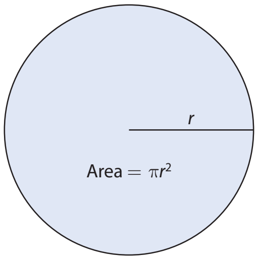
17. $c = 0.5\sqrt{a} + 0.2$; $C = 0.5\sqrt{a} + 6.2$ **The total cost with installation would be \$13.13**

17. Multi-Step A framing store uses the function $\left(\frac{c - 0.2}{0.5}\right)^2 = a$ to determine the total area of a piece of glass with respect to the cost before installation of the glass. Write the inverse function for the cost c in dollars of glass for a picture with an area of a in square centimeters. Then write a new function to represent the total cost C the store charges if it costs \$6.00 for installation. Use the total cost function to estimate the cost if the area of the glass is 192 cm². **See above.**

18. Make a Conjecture The function $f(x) = x^2$ must have its domain restricted to have its inverse be a function. The function $f(x) = x^3$ does not need to have its domain restricted to have its inverse be a function. Make a conjecture about which power functions need to have their domains restricted to have their inverses be functions and which do not. **A power function whose power is even will have to have its domain restricted. A power function whose power is odd does not have to have its domain restricted.**

Lesson Performance Task

One method used to irrigate crops is the center-pivot irrigation system. In this method, sprinklers rotate in a circle to water crops. The challenge for the farmer is to determine where to place the pivot in order to water the desired number of acres. The farmer knows the area but needs to find the radius of the circle necessary to define that area. How can the farmer determine this from the formula for the area of a circle $A = \pi r^2$? Find the formula the farmer could use to determine the radius necessary to irrigate a given number of acres, A. (Hint: One acre is 43,560 square feet.) What would be the radius necessary for the sprinklers to irrigate an area of 133 acres?



$r = \sqrt{\dfrac{43{,}560A}{\pi}} \approx 118\sqrt{A}$

The radius necessary to irrigate an area of 133 acres would be about 1361 feet, or a little over $\frac{1}{4}$ of a mile.

Extension Activity

Explain to students that the farmer is trying to conserve water and is limited to 3 million gallons of water for each irrigation. If the crops need 2 inches of water per irrigation to be productive, have students calculate the maximum radius of the circular area the farmer can irrigate. (Hint: 1 gallon is approximately 0.13 cubic feet.) The volume of water is given by $V = \pi r^2 d$, where d is the depth of water. Solving for the inverse function gives $r = \sqrt{\dfrac{V}{\pi d}} = \sqrt{\dfrac{3{,}000{,}000 \cdot 0.13}{\pi \cdot \frac{2}{12}}} = 863$ feet.